M. B. W. TENT

Emmy Noether

The Mother of Modern Algebra

Emmy Noether

The Mother of Modern Algebra

Emmy Noether

The Mother of Modern Algebra

M. B. W. Tent

A K Peters, Ltd.
Natick, Massachusetts

Editorial, Sales, and Customer Service Office

A K Peters, Ltd.
5 Commonwealth Road, Suite 2C
Natick, MA, 01760
www.akpeters.com

Library of Congress Cataloging-in-Publication Data

Tent, M. B. W. (Margaret B. W.), 1944-
Emmy Noether : the mother of modern algebra / M.B.W. Tent.
p. cm.
Includes index.
ISBN 978-1-56881-430-8 (alk. paper)
1. Noether, Emmy, 1882-1935--Juvenile literature. 2. Mathematicians--Germany--Biography--Juvenile literature. 3. Algebra--Juvenile literature. I. Title.
QA29.N6T46 2008
510.92--dc22
[B]

2008021713

Cover image: Courtesy of Peter Roquette.

Printed in India

12 11 10 09 08 10 9 8 7 6 5 4 3 2 1

To all the bright mathematics students,
past and present,
whom I was privileged to teach at
the Altamont School

Contents

Preface

This is the life story of Emmy Noether, the most important female mathematician who ever lived. In 1973, Irving Kaplansky wrote " … it is surely not much of an exaggeration to call her the mother of modern algebra," and his assessment still stands. Because no one expected Emmy to grow up to be an important scientist, the records of her early life are sketchy. After all, it was assumed that she would grow up to be a wife and mother. Instead, she was a genius who chose her own distinctive path. However, we should forgive her parents for not foreseeing her remarkable future given the limited options available to women at the time.

The story that I have written is based on the scraps of information that we have, but of themselves those tidbits do not present a coherent story. I wove the story of her life around the events that appear in the oral and written records, fleshing out the story with what I know of life in Germany at the time and what I know of how bright children explore mathematics. In other words, although this is a biography of Noether, it has an element of fiction as well. I hope her irrepressible charm and keen intellect come through.

In the story, I have incorporated a few German words and expressions, which appear in the glossary at the end of the

book, along with explanations of mathematical terms. When you encounter words that you don't know, you should consult the glossary.

There are some tantalizing hints of Emmy Noether's childhood that are part of the historical record. For example, we know that she went to a birthday party where the children were asked to solve puzzles that only Emmy was able to solve. We don't know whose birthday it was, how old Emmy was, or what puzzles were presented. We know that the family had musical evenings in their home in which a guest played the violin and Emmy's mother played the piano beautifully, but we don't know that they played Brahms's Sonata in G Major, op. 78, for violin and piano or who the violinist was. This sonata would have been current at that time and would have fit in to an evening in a home setting, and my great aunt took great pleasure in playing this sonata in 1918. We know that the only piano piece that Emmy could play well was "The Happy Farmer." We know that Emmy's mother complained about the water bill that they had to share with the Wiedemann family downstairs, but we don't know exactly what she said, or how Emmy's father responded. We don't know specifically what games Emmy and her brothers played, but Nine Men's Morris has been played throughout Europe for many centuries, and it is still a popular game with German children. (The set-up and the rules for the game are in the appendix at the end of this book.) Since we know none of the details of these and other events, my only option was to construct plausible scenes of her childhood in a way that I think they might have happened, working with the facts that are available.

The records of her activities as an adult are better documented, but even there the details are scanty. Since no one knows exactly what people said in private conversations, I have created dialogues as I think they could have happened. However, we do know that in 1915, when Emmy was denied her *Habilitation* on her first try, Hilbert asked his colleagues (all professors) if they realized that it was a university—not a bathing establishment—that they were talking about, and the professors were scandalized. They were unwilling at that time to bestow the title of professor on a mere woman. We also know that for years Emmy went swimming every day with Nina Courant and that the water was often muddy. We do not know that Emmy cheered as her students kissed the goose girl in the city hall square, but I think she must have. That would have been decidedly Noether-like.

Emmy Noether's experiences facing discrimination form a central part of her story. As a female and a Jew, she encountered daunting obstacles. I do not know that her father told her the stories of Hypatia, Kovalesky, and Young, but I think it is possible, and the parallels are poignant. Young women of today do not face the obvious barriers that Emmy Noether and others faced, yet they still must contend with stereotypes and the many demands and limitations on women in our society. I hope with Noether's example, a few more young women will discover the magic of mathematics.

I became very fond of Emmy Noether as I constructed her story—each time I read the tributes at the end of her story, I am moved once more. She was warm and lively, utterly selfless, and became passionate about mathematics as she matured. She gives new meaning to the expression "pure mathematics."

Acknowledgments

Although I had wonderful help from many people as I prepared this book, I should start by thanking Klaus Peters at A K Peters, Ltd., who originally suggested the project to me. It was an excellent idea, and I have had fun doing it. I have long worried about why I, as a math teacher, couldn't convince more girls to go on to study higher mathematics, and writing this book seemed like a concrete step in that direction.

Next, I would like to thank Dr. Heinrich Hirschfelder and his wife, who helped my husband and me so generously as I began my research in Erlangen in June 2006, and even entertained us regally for coffee and a wonderful meal in their garden. Dr. Hirschfelder showed us around Erlangen and introduced us to his brother who located an antique algebra textbook in the university library from which Emmy probably first learned algebra with her father. Dr. Hirschfelder also expedited my research in the Erlangen *Stadtarchiv* and has continued to provide me with materials from his research into the history of Erlangen. It would have been difficult to complete this project without his help. I also want to thank Barbara Fischer at the Erlangen *Stadtarchiv* for her patient and competent help.

Also in Erlangen, Laura Baxter located a copy of Ilse Sponsel's work on the Jewish cemetery in Erlangen, which provid-

ed the only concrete information I have found on Emmy's youngest brother Robert. I met Laura through my neighbor Jane Spann in Birmingham, Alabama, and I am most grateful for their help.

I want to thank several people in Göttingen. Dr. Axel Wittmann's help was invaluable once more: introducing me to people, arranging meetings, providing me with resources, and escorting my husband and me to Dr. Cordula Tollmien's lecture on Emmy Noether at the Mathematical Institute on June 13, 2006. Dr. Tollmien's excellent talk that evening helped me as I started on this project. It was she who suggested that Emmy probably would have used a Dr. Oetker mix to make her pudding—I hadn't realized that such packaged foods were available in the 1920s. I also appreciate the help of the firm Dr. August Oetker Nahrungsmittel KG in providing me with a picture of the pudding package from 1922 and allowing me to use it. Thanks also to Helmut Rohlfing at the University Library at Göttingen for providing photos from the library's collection and for his permission for me to use them.

Dennis Wittmann took several photographs at the Mathematical Institute in Göttingen for me. His eye is good and the quality of his photos is excellent. Thank you, Dennis.

I also greatly appreciate Professor Israel Kleiner's confirmation that Irving Kaplansky was probably the first person to call Emmy Noether "the mother of modern algebra" in his paper on commutative algebra in 1973 in the report of the 1972 *Conference on Commutative Algebra*, published by Springer-Verlag on page 155: " … it is surely not much of an exaggeration to call her the mother of modern algebra."

I made extensive use of two books that have been published on the life of Emmy Noether. Auguste Dick's *Emmy Noether 1882-1935,* published in 1981 by Birkhäuser in Boston, and James W. Brewer and Martha K. Smith's *Emmy Noether: A Tribute to Her Life and Work*, published in 1981 by Marcel Dekker, Inc. in New York. Both provide a compilation of the writings about and tributes to Emmy Noether. The quotations in the last two chapters of my book are found in both those volumes.

Drs. Sabine and Christian Koch in Berlin have once again contributed to this project, providing helpful suggestions over the years, as well as wonderful hospitality over and over again. I thank them most heartily.

I want to thank the station master of the Berner Oberland Bahn at Zweilütschinen, Switzerland, for allowing me to photograph Swiss railroad cars from the 1930s.

I also want to thank my father, Raymond Wyman, for posing in the picture of the cat ladder in an Alpine village and for introducing me to the town of Wengen many years ago. My brother John Wyman and I enjoyed walking up the slopes of the Eiger from Wengen, where we, like Emmy Noether, found ourselves out of breath as we hiked the steep uphill path beginning at 1250 meters above sea level. I thank John for humoring me in that.

The Altamont School in Birmingham, AL, was intimately involved in the preparation of this book. The mathematics department was supportive as I juggled teaching classes with writing. The student body was helpful as they listened to my lectures on great mathematicians over the years. In the fall of 2006, they experimented most effectively with the German

academic tradition of shuffling their feet before and after I gave an illustrated lecture on Emmy Noether. Jennifer Grissom was so accommodating as to serve "Emmy Noether's Famous Pudding" in the lunchroom the day of my lecture. The math team gamely learned to play Nine Men's Morris, which they continued to happily play whenever I allowed it. I appreciate Anna Stalker's willingness to read the manuscript at the same time that she was considering whether she wanted to follow Emmy Noether's footsteps to Bryn Mawr. Faculty members Mary Martin, Caroline Collins, and Sim Butler all gave me excellent feedback on the manuscript as well. Mary Martin was also willing to practice Nine Men's Morris with me. Lizanne Gray has made some wonderful photographs for me, including the picture of the Nine Men's Morris game that appears in this work. She is a skilled photographer with whom I feel privileged to work.

Mary Gray Hunter, Altamont's talented photography teacher, took the portrait of me that appears on the jacket of this book. She made me look reasonably attractive, and she did it with grace. Thank you, Mary Gray.

I also appreciate Catalina Herrera's careful and intelligent editing.

Eve Graham was especially helpful as I constructed the story of Emmy Noether's youngest brother Robert, explaining to me how a mentally disabled child would develop. As a professional psychometrist, she was able to provide me with specifics that I couldn't have known otherwise.

My editors at A K Peters, Charlotte Henderson and Ellen Ulyanova, as well as Carolyn Artin and Christine Horn were able to help me transform a rough manuscript into a book that

I hope is a good finished project. Along with Klaus Peters, they helped me clarify some of the difficult mathematics.

I especially thank my family for supporting me in this project. My children, John and Virginia, read the manuscript carefully and made timely suggestions. John picked up on historical inconsistencies, and Virginia focused on the human aspects of the story. When they were school children in Hannover, Germany, in 1983, we all learned to play Nine Men's Morris in the city park. I also could not have completed this project without help from my husband Jim, who found himself teaching me 1920s and 1930s German history over dinner many evenings and editing sections of manuscript from time to time. He always protests that he doesn't know math so he can't help me, but his help was invaluable, as always. His support of me in my early retirement from teaching has made this whole project much easier.

There are certainly many other people whom I have neglected to mention who helped me on this project, and I apologize for not mentioning them all. This was a huge project, and I could not have done it without help from friends and family.

Part I
Childhood

Anna's Birthday Party

"Emmy, are you ready to go to Anna's birthday party?"

Emmy's mother was impatient. "It's time to go. But wait! Emmy, why are you wearing that old green dress? No, no, my child! That's not your prettiest dress. Come along! Let's get out your blue dress with the lovely embroidered collar. You'll be much more attractive in that dress." Emmy Noether was 11 years old, chubby and awkward, and not particularly pretty. Many years later, the mathematician Hermann Weyl observed that the Graces did not preside at Emmy's cradle. No one ever described her as beautiful.

"I thought this one looked all right," Emmy protested. "You always say we should be frugal, and this collar was clean just yesterday. I was thinking about how long Elsa spent starching and ironing it and making it look just right, and I didn't spill anything on it. See? It's nice and clean."

"For a party, you should wear a party dress," her mother explained. "Be a good girl and put on your best dress. You can be sure that all your friends will be dressed in their best clothes as well."

Emmy did as her mother said and changed her dress.

"There, now, that's better. You look quite presentable. Come along quickly or we'll be late." Together Emmy and her mother walked the four blocks to Anna's house.

At the party, the children played games and sang, and then Dr. Herder, Anna's father, said he had some puzzles for them to solve. Silence fell as everyone waited to hear the first puzzle.

He began: "A sculptor named White, a violinist named Black, and an artist named Red met in a café. One of the three said, 'I have black hair, one of you has red hair, and the other has white hair, but none of us has hair the color that matches his name.' Mr. White answered him, 'You are quite correct.' Now I ask you, children, what color hair does the artist, Mr. Red, have?"

This was a difficult puzzle! Surely Dr. Herder had not given them enough information to solve it!

"Dr. Herder!" one of the children called out. "Don't you need to tell us the name of the first man?"

"No," he answered. "I have told you enough."

The room grew quiet as the children started to think the puzzle through. Then suddenly Emmy shouted out: "I know! The artist has black hair!"

The other children looked at her in astonishment. "How do you know?" they all asked.

"She is correct," Anna's father said. "Can anyone tell me how Emmy might have figured it out?"

There was silence. All the other children were baffled. Anna's father urged them, "Come on, try a little harder! I'm sure you can at least start to solve it. Think about the facts you know."

Still there was silence. Finally, Anna whispered to her friend, "Emmy, how did you know his hair is black?"

Dr. Herder asked the group, "Would you all like Emmy to explain?"

Everyone chorused, "Yes!"

Emmy began, "Well, Mr. White answers the man who has black hair, so that means Mr. White does not have black hair, and we know he doesn't have white hair either (since his name is White), so Mr. White's hair must be red. That leaves only black and white hair for Mr. Black and Mr. Red, but Mr. Black can't have black hair (since his name is Black), so his hair must be white. That means that Mr. Red's hair must be black. That was fun! Do you have another puzzle for us, Dr. Herder?"

"That was too hard!" the other children complained.

"No, it wasn't," Dr. Herder replied. "You could have figured it out, but you gave up too quickly. Here's another puzzle for you. Let's see if someone else can get the answer this time.

"Let's say that you want to write all the whole numbers from 1 to 100 on a large slate. How many different numbers will have at least one digit that is a 7?"

"I know! I know!" Emmy called out.

"Please wait a minute, Emmy. I'd like to give the others a chance."

A few minutes later, Josef volunteered, "Dr. Herder, I know!"

"Good for you, Josef. How many numbers are there?" Dr. Herder asked.

"There are 20 of them—20 numbers," Josef said.

"How did you figure that, Josef?" Dr. Herder asked.

"Well, there are ten numbers with 7 in the ones' place: 7, 17, 27, 37, … all the way to 97. Then there are ten numbers in the 70s: 70, 71, 72, … all the way to 79. That gives a total of 20 numbers."

"Emmy, do you agree with Josef?" Dr. Herder asked.

"No, because Josef counted the number 77 twice, once for the ones' place and once for the tens' place! There are only 19 different numbers with 7 in them," Emmy announced.

"You are correct, Emmy! Very good! Josef, do you see your mistake?"

"Yes, Dr. Herder," Josef agreed. "I didn't think of that."

"All right, then, children, here is my last puzzle: There is a man who wants to take a wolf, a goat, and a cabbage across the river. His rowboat is so small that he can carry only one of them at a time. If he takes the wolf across first, the goat will eat the cabbage while he is gone. If he takes the cabbage across first, the wolf will eat the goat while he is gone. How can he carry the wolf, the goat, and the cabbage across the river without losing any of them? Now, Emmy, please wait a bit and give the others a chance to figure it out."

There was silence.

Once again, the other children were baffled. Finally, Dr. Herder looked at Emmy. "Do you know how he can do it, Emmy?" he asked.

"Yes," she answered eagerly. "Shall I explain?"

All the other children said yes. "It's really quite easy. First, the farmer should take the goat across the river, leaving the wolf and the cabbage alone since a wolf won't eat cabbage. Then the farmer leaves the goat on the other side and goes back for his next load."

"But then no matter what he takes next, either the goat or the cabbage will be eaten!" Anna cried.

"Give me a minute. I thought of that," said Emmy. "So the farmer goes back and gets the cabbage, but now comes the clever part. When he leaves the cabbage on the other side of the river, he puts the goat back into the boat and carries it back to the first side again, so that only the cabbage remains on the second side. Now he leaves the goat on the first side of the river again and carries the wolf over to join the cabbage on the far side. The wolf won't eat the cabbage this time either. On his last trip, the farmer takes the goat one last time, and in that way he gets all three to the other side. It may seem indirect to take the goat across the river three times, but the farmer has to be careful. You see, the goat is really the only problem—he is the one that will either eat or be eaten. As long as the goat isn't left alone with one of the others, everything is fine."

"That is very good, Emmy," Dr. Herder said. "You thought it through carefully and well, but can anyone think of a different way that it could have been done?"

Once again there was silence. After a few minutes, Dr. Herder turned to Emmy. "Emmy, can you think of another way?"

"Yes," Emmy quickly said. "After he took the goat across the first time, he could have taken the wolf next and left him on the other side instead of the cabbage, again carrying the goat back and leaving the wolf instead of the cabbage alone on the far side. Then he would have brought the cabbage across for the wolf not to eat, and finally he would have taken the goat. It would have worked exactly the same way."

"You're right, Emmy," Dr. Herder congratulated her, "and now I believe it is time for the birthday cake!"

After her guests had gone home, Anna sat talking with her parents. "Father," she asked, "how did Emmy solve those puzzles so quickly?"

"She did very well, didn't she?" Dr. Herder agreed. "I suspect she is a very bright child. Her father is a prominent mathematician at the university, you know. She appeared to enjoy the puzzles greatly."

"Can she be a mathematician like her father when she grows up?" Anna asked.

"I doubt it, Anna. There is no place at the university for a woman scholar," her father said. "Emmy will grow up to be a wife and mother. That's a full life for a woman."

The Girls' School

Emmy had entered the *Höhere Töchterschule* (privileged daughters' school) in 1889 when she was seven years old, joining approximately 140 other girls, daughters of prosperous doctors, lawyers, businessmen and academics, all of whom were being prepared for future lives as wives and mothers. Although girls in the 1800s were not expected to become scholars, they needed to learn to read, write, and add. Also, as cultivated women, they needed to be able to speak French and English, to play the piano acceptably, and to cook and sew. The brothers of these same girls went to the *Grundschule* (elementary school) and then on to the *Gymnasium* (high school), a more demanding program for boys who would grow up to be important in the town and in society. The general view at the time was that girls didn't need a serious education; they simply needed to be "finished."

Emmy walked to school with her friends every morning. Although she was hardly the prettiest girl in the class, everyone agreed that she was jolly company. She did well in school, and she was a capable, cheerful girl. If she was sometimes a little too loud—perhaps even boisterous—she did everything with such gusto and good humor that no one was offended for long. And, above all, she was kind—the thought of anyone suffering was abhorrent to her.

Höhere Töchterschule in Erlangen (Courtesy of Stadtarchiv Erlangen)

One morning during recess, Emmy noticed that her friend Grete was upset. "Grete!" Emmy ran up to her clearly unhappy friend. "What's the matter?"

"They won't let me play! They hate me!" Grete sputtered.

"Why won't they let you play?" Emmy asked.

"I don't know!" and Grete burst into tears.

"Come on, Grete," Emmy said putting her arm around Grete. "Let's go play our own game. I don't like the way they are playing."

"Thank you, Emmy," Grete said quietly. "You really are my friend, aren't you?"

"Of course! I'll always be your friend," Emmy reassured her. "Let's play hop-scotch! Come on!"

It wasn't long before the other girls came over and wanted to play with Emmy and Grete. By the time recess was over, they were all friends again.

German schools have always included weekly religious education as part of their program. The parents have a choice of Roman Catholic, Protestant, or Jewish instruction, all of which are usually taught by a member of the respective clergy. The nine Protestant girls in Emmy's class joined the Protestant girls from other classes for their religious lesson with Dr. Seeberg. At the same time, the only Catholic girl in Emmy's class joined the Catholic girls from other classes for their catechism with Chaplain Müller. Because Emmy's family was Jewish, she went with two other Jewish girls from her class to the Jewish study group with *Herr* Morgenthau. By the time Emmy was 15 years old, she was the only Jewish girl out of a total of nine girls remaining in her class.

Several generations earlier, following the policy of the Grand Duchy of Baden, the Noether family had exchanged their traditional family name of Samuel, a name that came straight out of the Old Testament, the sacred text for Judaism, and marked the family as Jewish. The name Noether was clearly not Jewish, allowing the family to be assimilated into the general population. Earlier generations of the family had been involved in the hardware business, but Emmy's father and grandfather had studied at the university. By the time Emmy started school, her father was a well-respected scholar and professor of mathematics at the University of Erlangen in southern Germany. As a child, Emmy never experienced anti-Semitism although in the recently united Germany under the Kaiser there had always been an undercurrent of anti-Sem-

itism. However, her family was prominent in the community, and their religion was not an issue.

Every day after school, Emmy and her younger brothers Alfred and Fritz arrived home in time for dinner in the early afternoon. Her mother and little brother Robert were always waiting for them. As Emmy set the table and Alfred and Fritz washed their hands, they heard the familiar ker-thump, ker-thump of their father, Professor Dr. Max Noether, as he slowly climbed the long flight of stairs to their second-floor apartment. When Emmy's father was 14 years old, he had been stricken with polio, a dreaded disease that often crippled a child for life. Although it had limited Max Noether's activities as a child and he had been lame ever since, it had not prevented him from leading a productive, scholarly life. His steps may have been plodding, but there was nothing slow about the pace of his mind. His children, who also knew how important their father was, adored him. They were growing up in a warm, supportive, intellectual home environment.

Ida Noether, Emmy's formidable mother, was bustling back and forth between the kitchen and the dining room, trying to work around little Robert, who seemed to be in the way no matter which way she turned. She had been worrying about him lately. Although he was five years old, he had been late learning to walk—he had been well over two years old when he took his first tentative steps—and even now he walked with a strange gait, awkward and jerky, and he always seemed to be unsure of where his feet were. His conversation, if you could call it that, was more like that of a small child,

putting together two or three words with difficulty instead of speaking the whole sentences you would expect of a five-year-old. Mostly, he just stood and looked at everyone blankly. The other children had tried to teach him how to kick a ball, but Robert couldn't do it.

There was no time for such worries now, however. Emmy's mother brought out a pot of stew and a large loaf of bread (fresh from the baker that morning) that Elsa, the kitchen maid, had just sliced for dinner. A large pitcher of delicious, steaming milk was already on the table, nice and hot, just the way they liked it on a cold, gray day.

As they sat down to dinner, Emmy's mother complained to her husband, "Max, when I was making dinner today, Elsa and I noticed that *Frau* Wiedemann downstairs was running water the whole time. There was hardly enough water for me to make the stew! It's outrageous!"

"Now, Ida," her husband said, "the Wiedemanns need water too. You shouldn't be so hard on them."

"Yes, they do, but how much do they really need?" *Frau* Noether demanded.

"They have several children, and they certainly need water," Professor Noether said. "They also have a new baby, you know."

"Of course I know that!" she snapped. "That baby screams all the time. I wonder that he has any voice left!"

"Don't you suppose that they are sometimes annoyed by us, too?" Professor Noether asked mildly. "After all, we live directly overhead. Sometimes we must sound like a herd of elephants above them."

"I don't think they can hear us over all their noise!" she answered. "I am dreading the water bill that will be arriving

any day now. The Wiedemanns must use twice as much water as we do, but we still have to split the cost evenly. I watch carefully how much water Elsa uses as she cleans up the kitchen. This joint water bill is grossly unfair, if you ask me."

"I don't think it will send us to the poor house," Professor Noether commented wryly. Changing the subject, he turned to his daughter and asked, "Emmy, what did you learn in school today?"

"In French class, we learned '*Maître Corbeau, sur un arbre perché* …,' the fable about the fox and the crow." Emmy said. "It was fun! The fox was mighty clever! He tricked that crow into dropping his cheese right where the fox could pick it up! 'Mr. Crow, if your voice is as pretty as your feathers, you must be the most impressive bird in the forest!' Yes, that lesson was worth a cheese, wasn't it Papa?"

"Yes, Emmy," he said, "I have always liked La Fontaine."

"Who is he?" Emmy asked her father.

"He was the Frenchman who wrote the poem, silly girl! If you like the poem, you should at least know the name of the person who wrote it," her father chided.

"Is he still alive?" she asked.

"No, no. He died 200 years ago, so you are not likely to meet him on the streets of Erlangen or even of Paris, but he did write some wonderful stuff. Are you going to learn to recite the poem?" her father asked.

"Yes, Papa. I already know the first few lines," she proudly announced.

"Alfred and Fritz, have you learned '*Maître Corbeau*' yet?" asked their father.

"No, Papa. Are crows really that important?" asked Fritz. "I like arithmetic."

"Well, of course, we all like arithmetic," the mathematician agreed, "but we can't expect to do only mathematics in our lives. That would make for a rather limited life. We all need music and poetry and history and art for balance."

"Yes, indeed," *Frau* Noether interjected, "Emmy needs to start making some progress with the piano sometime very soon."

"Emmy, is the piano not coming along so well?" her father asked gently.

"No, Papa," she admitted. "I don't like it very much. When Mother plays, it's beautiful, but the keys just don't sound the same when I play them. Maybe I need to watch her more carefully when she plays."

"That is certainly not the answer!" her mother said. "The way to learn to play the piano well is to practice. When I hear you practice, Emmy, you spend perhaps 20 minutes, playing one line and then another, never perfecting anything. When I was your age, I practiced the piano seriously for an hour a day. The only time you do that is if I stand over you and force you, and it always ends the same way: we both get frustrated." Turning to her husband, she continued, "Max, I wonder if Emmy will ever master the piano."

"Well, if she doesn't, it won't bother me too much!" Professor Noether said. "I expect Emmy will be good at other things."

"She still needs to learn to play the piano well," her mother said firmly. "She plays "The Happy Farmer" acceptably, but that is hardly serious music! Her work on Bach's Gavotte from the French Suite no. 5 is appalling. She struggles through the first line and then repeats it only a little more successfully, but

then she stumbles and cannot complete the second line. I don't know, Max. I just don't know."

A Proper Upbringing for a Girl

"Mother, I've finished a row in my sewing," Emmy announced. "May I go out and play?"

"Let me see your work, child. Oh. Yes, I see. You have done a row of stitches, but the row is not straight, and some of the stitches are too long. See, the stitches are quite uneven. Look, dear, you're holding the needle wrong. You have to hold it between your thumb and finger, like this," her mother showed her.

"But Mother, isn't that what I was doing?" Emmy protested.

"No, Emmy, that is not what you were doing," her mother corrected her. "You should hold the needle like this. Yes, that's better. Now, I want you to try again. Maybe if I use a pencil to draw a line on the fabric you will be able to sew straight."

"Why do I need to learn to sew anyway?" Emmy asked.

"Because a girl must learn to sew," her mother explained. "When you are grown, you will need to be able to sew well. I don't expect your work to be perfect now—that will come later—but you must start working toward that goal now. Here is a good, straight line for you. As you sew, please make sure your stitches are all on my pencil line."

"But Mother, Papa doesn't sew," Emmy observed as she sewed.

"No, of course your father doesn't sew! He is a man. Men do not sew. You will grow up to be a woman, and that means you must learn to sew," her mother said.

"Yes, Mother," Emmy said as she carefully made three more stitches—all the same length—along her mother's line. Then she continued, "Mother, I've been thinking. Does that mean that Alfred and Fritz and Robert won't have to learn to sew?"

"Of course they won't learn to sew!" her mother answered. "They are boys. They go to school and learn important things."

"But I go to school too," Emmy protested.

"Yes, dear, you go to the *Töchterschule* and learn the things that a girl needs to know," her mother explained.

"Why can't I go to the real school—to the *Gymnasium*—like Alfred and Fritz?" Emmy asked.

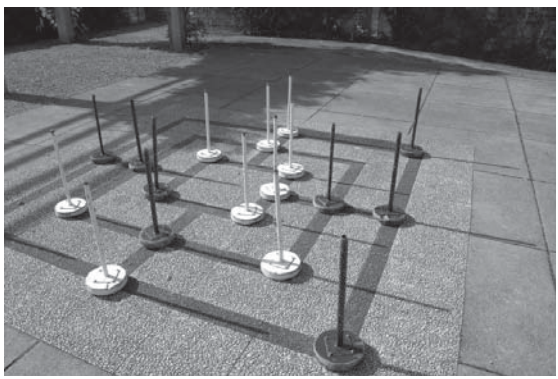
"Emmy, you're a girl. Girls just don't do that," her mother said.

"I don't understand why not," Emmy pouted. "Mother, may I please go outside and play now? I'm tired of sewing!"

"All right, you may go out for a little while, but later you must continue with your sewing, and then you must practice the piano."

"Yes, Mother," Emmy sighed.

Outside, Emmy waited while Fritz and Alfred finished their game of Nine Men's Morris (see the appendix for rules of the game) on a grid they had scratched into the dirt many times over the years. They all agreed that Emmy would play



Nine Men's Morris board in a park (Courtesy of Lizanne Gray)

the winner. When the boys' game was over, Emmy called out, "Okay, Alfred! Now it's my chance to play! I'll take the sticks and you take the pebbles. You go first."

"Okay, Emmy!" Laying a pebble down carefully on the grid, Alfred said, "It's your turn!" Emmy and Alfred took turns, placing their sticks and pebbles on the grid, each trying to make a mill (a set of three tokens in a straight line) but also trying to keep their opponent from making a mill. Once all the pieces were on the grid, they took turns moving one piece to an adjacent spot on the grid, as they tried to make further mills. Each time they completed a mill, they could remove one of their opponent's pieces.

It took only a few turns for Emmy to be ahead as she carefully planned her moves, systematically removing Alfred's pebbles from the grid. Suddenly she stopped. "Alfred, I didn't mean to make you lose!" Emmy admitted. "You made your mistake at the very beginning, and I should have warned

you. I'm sorry! Let's play again, but this time I'll try to help you."

"I can help him," Fritz volunteered. "I messed up in the first game, but I'm a pretty good player. Why don't we play as a team against Emmy, Alfred? Wouldn't that be more fun?"

"Do you mind if we do that, Emmy?" Alfred asked.

"No, I don't mind," Emmy agreed. "That would be more fun for everyone. You go first, because that will give you the advantage."

This time the game took considerably longer because each time after Emmy played, the two boys had to consult about their strategy, and then Emmy had to comment on their decision. In the end, neither side could break the other's remaining mill and nobody won. "You boys played a good game," Emmy congratulated them. "Shall we play again?"

Just then Fritz's friend Hans Falckenberg arrived. "Who's winning?" Hans wanted to know.

"Well, I won the first game," Alfred explained. "Then Emmy beat me, and then Fritz and I played as a team against Emmy and it ended in a draw. Do you want to play, Hans?"

"Sure!" Hans answered. "How about Fritz and me against Emmy and Alfred?"

When she went back inside, Emmy sat glumly at the piano. She tried a bit of Bach. Plink, plank, plunk! Once again, she had trouble getting beyond the first line of the gavotte. "Well, there's no harm in going back to the music I can play," she said to herself as she started her favorite piece, "The Happy Farmer."

"Emmy, is that 'The Happy Farmer' again?" her mother called from the kitchen.

"Yes, mother," Emmy answered. "I'll return to Bach in a minute."

"Now would be a good time to do that!"

As Emmy tried again, her mother strode into the room to watch Emmy play for a few minutes. "No, no!" her mother shouted. "Look, Emmy, your fingers are too flat. You must curve them gracefully. You should play only with the tips of your fingers. Hold your hands like this. Yes, that's better. Now, play a scale for me, slowly and smoothly. Yes. Yes, that's much better. How about one more try on the Bach?"

Emmy played the first chord of the gavotte, and then she stopped. "Mother," she began, "is something wrong with Robert? When Alfred and Fritz were his age, didn't they talk and play and make jokes and things? They could play Nine Men's Morris by his age, couldn't they?"

"Yes, Emmy, I think they could," her mother answered. "That's a difficult question. Robert isn't like you and Alfred and Fritz."

"He acts like the Wiedemanns' three-year old downstairs," Emmy said. "I think Alfred and Fritz wanted to play games with me when they were his age, but Robert just stands there."

"Yes, you're right that he's a little slow. We'll need to figure out what to do for him, but for now I think we'll just care for him as well as we can. When he's older, he may be able to go to the *Grundschule* and learn a few basic things, and then maybe he can go on to the vocational school. Later on he might be able to work in the stationery store down the street."

"Wouldn't it be better for him to go to the *Gymnasium*? Isn't he smart enough to learn things?" Emmy asked.

"I don't know," her mother said. "I don't know what will be best for him. We'll have to wait and see." Emmy could see that her mother was upset.

Then taking hold of herself, her mother continued on a more cheerful note, "Now how about a little more Bach? When *Tante* Elisabeth comes next week, I would like you to be able to play the gavotte for her."

"Is *Tante* Elisabeth coming next week?" Emmy asked with delight.

"Yes, and she will stay for about two weeks. I need to do some serious house cleaning, and she is going to help me," her mother explained. *Tante* Elisabeth was *Frau* Noether's youngest sister, and all the Noether children enjoyed her visits greatly. She liked to play games with the children, and she loved puzzles. When she came for a visit, she usually brought each of the Noether children a good, challenging puzzle, and then worked with them, helping them solve the puzzle.

That evening, the Schmidts joined the Noethers for supper and a musical evening. Supper was a simple array of bread, cheese, and sliced sausage, topped off with a large bowl of fresh fruit, but the real entertainment for the evening was music.

As soon as they finished eating, *Herr* Schmidt took out his violin, sat down at the piano, and began to tune his violin carefully. His young son Tomas watched with great interest. "Father, why do you turn that thing, and then pluck a string, and then turn it again?" Tomas asked.

"I'm trying to make it so that it is in perfect tune with the piano," *Herr* Schmidt explained. "Music, no matter how well it is played, is wretched unless the instruments are in tune. *Frau*

Noether's piano is always perfectly tuned, and my violin must match it."

As soon as *Frau* Noether and Emmy finished cleaning up the supper dishes, *Frau* Noether walked briskly into the living room and sat down at the piano. "What shall we play tonight, *Herr* Schmidt? What have you been playing lately?"

"Well, you know, I have been spending some time on Brahms's Sonata in G Major. It is really quite lovely!" he said. "I have the music right here."

"I remember this!" *Frau* Noether said as she looked at the first few pages. "Yes, I played this several years ago when it first came out. I love Brahms. His music is so lyrical. I think I remember it well enough to play it." As she firmly played the first chords, *Herr* Schmidt started playing, making the violin sing in perfect balance with the sweet chords from the piano. Then, suddenly, the piano took off, with brilliant broken chords—Emmy wondered if those were what her piano teacher called *arpeggios*—while the violin played a steady, somewhat lilting tune. By this time, *Frau* Noether's fingers were moving so fast that Emmy could hardly see them.

Suddenly *Frau* Noether stopped. "*Herr* Schmidt! I had forgotten how beautifully the piano and violin are intertwined in this piece! And you are playing it beautifully, I must say. Your violin truly sings to the piano."

Herr Schmidt replied, "But *Frau* Noether, it is hard to believe that you have not played this piece in several years! It is as if you have been practicing it every day for a week. You read music skillfully and beautifully!"

As the two musicians started again from the beginning, Emmy was lost in thought. She knew her mother played the

piano beautifully, and she had to admit that the music this evening was enchanting, but she couldn't imagine spending the time necessary to learn to play either the violin or the piano that well. No, she could appreciate music when other people played it, but she simply wasn't interested in learning to control her fingers to produce the lovely sounds her mother's fingers could. Yes, it was beautiful, but no, she had no desire to learn to play the piano as well as her mother could.

She looked over at her father, whose eyes were closed as he listened. Was he humming along? Yes, she thought he was. So, perhaps she could enjoy the music, even take great pleasure in it, without producing the actual music herself. She decided that she would enjoy her music that way—the way her father did—although she knew she would have to suffer through several more years of piano lessons regardless. Her mother was a powerful personality with a clear idea of what kind of woman Emmy should grow up to be.

"Papa," Emmy said the next evening as she snuggled beside him on the couch in the glow of the gas light on the wall, "what do you do all day?"

"What a funny question from my little girl!" he said. "Well, I am a mathematician. I spend my days doing mathematics and teaching mathematics."

"I don't know what mathematics is," admitted Emmy. "Would you tell me?"

"Well, you are learning arithmetic, and that is the beginning of mathematics. You will need to learn it in order to keep track of money and to plan your sewing and cooking."

"But is that what you do all day?" Emmy asked.

"No, no, Emmy. That isn't what I do!" her father laughed. "A mathematician works with abstract ideas. I hope when Alfred and Fritz grow up that at least one of them will be a mathematician too," her father explained. "They are starting to learn algebra now."

"Couldn't I be a mathematician?" Emmy asked.

"No, Emmy," her father answered. "You are a girl. You could never be a mathematician. That is a life reserved for men. You will grow up to be a fine woman, like your mother, and you will spend your days cooking and sewing and bringing up children. You will have to leave serious academics to the men; that is the way it has always been."

"I don't see why I can't be a mathematician too," Emmy said.

What is Algebra?

Emmy continued her schooling at the *Töchterschule*, learning the arithmetic, geography, history, science, languages, and the rest that was required.

"Now, girls," *Fräulein* Jamin said as she began the French lesson for the day, "today we will learn more about limiting adjectives. This includes a review of numbers. You will find the list of the cardinal numbers—*un, deux, trois*, … [one, two, three, …], all the way to *un million* [one million]—as well as the ordinal numbers—*premier, deuxième, troisième,* … [first, second, third, …], all the way up to *millionième* [millionth]. You should already know most of them already. Does anyone have a question?"

There was silence in the room. "All right, then, we will continue with today's vocabulary for words like *year*, *month*, *week*, *day*, *dozen*, etc. Please look over the list. They are not difficult so I will expect you to know them all tomorrow. Now, the last part of the lesson concerns the multiplication tables. You already know them in German, so learning them in French should not present a problem. You can ask 'how much is seven times six' by saying '*combien font sept fois six*,' and the answer would be '*Sept fois six font quarante-deux*.' [Seven times six is forty-two.] Are there any questions?"

"Yes, *Fräulein* Jamin." Emmy raised her hand and stood up to speak. "Do we need to know all our times tables in French?"

"Yes, dear," *Fräulein* Jamin answered. "Are there any other questions? Good. Then right now I would like you to complete Exercise 9 on page 48 before the end of the class today, and tonight I would like you to do the exercise called 'Theme nine.' I will look for both of those in your notebooks tomorrow."

As she walked home from school that afternoon, Emmy thought about the French lesson. Although she liked *Fräulein* Jamin, she had to admit that the lesson today was dull. It wasn't hard; it was simply dull. She couldn't see how page after page of memorizing could possibly lead to speaking French, even if today it involved arithmetic. It was nothing more than rote learning. She wondered if *Fräulein* Jamin could actually speak French. She knew that *Fräulein* Jamin knew the words, but that wasn't the same as being able to speak the language. *Fräulein* Jamin taught English too, and she supposed there was no way that *Fräulein* Jamin could actually speak both languages. Could anyone do that? Probably not. Except, come to think of it, her father actually could speak French and English pretty well, and so could her mother. Maybe it was possible after all.

As a compliant child who generally did as she was expected, Emmy did well in school, learning English and French and arithmetic and other subjects. She occasionally questioned the teaching as she thought about it on her way home, and she wondered if she would ever master English and French, but she knew how to behave in school.

Once a week, her German teacher, *Herr* Bauer, worked with Emmy to correct her slight lisp. Her mother told her a lisp was cute in a small child, but must be corrected early, as it would not be cute in a grown woman.

"No, Emmy," *Herr* Bauer said. "You mustn't use your tongue to make the –*s* sound. If you put your tongue against your teeth it makes a sound like –*th*. That doesn't sound like a proper German –*s*. Look at me: you want to say *das*, not *dath*. Now, look directly at my mouth, and then look at your mouth in the mirror. I know you can correct this. You want to go *sssssss* like a snake, not *ththththt*. Now say *das*."

"*Dath*," Emmy said.

"No, Emmy, that's still not right. Look at your tongue. It should not be between your teeth. Your top teeth and bottom teeth should meet as you force the sound between them. Like this: *das*."

"*Das*," Emmy tried again.

"Yes, that's right, Emmy. Do it again."

"*Das*," Emmy repeated.

"Now say, *Was ist das*? [What is that?]"

"Was ist das?"

"Good, Emmy!" *Herr* Bauer announced. "Now, can you say *Das Haus ist gross* [The house is big]?"

"*Das Haus ist groth*," Emmy said slowly.

"No, Emmy," *Herr* Bauer said, "you got *das* and *Haus* right, but not *gross*. Remember it has to end in that same –*s* sound: *Das Haus ist gross*."

"Das Haus ist gross," Emmy said.

"Perfect!" Herr Bauer said. "You'll need to keep working on that sound. Any time a word has an –*s*, make sure your

tongue doesn't go against your teeth. You'll have to think about it until it becomes natural, but that shouldn't take too long. I know you can master this."

As she grew older, Emmy was invited out to an occasional dance party in town. She looked forward to these parties, but there was one problem: Although her friends enjoyed her company, Emmy was a terrible dancer. This didn't keep her from enjoying the dances, however! She loved to flounce around the floor, dramatically twirling and pointing her toe this way and that, but the result was comical rather than graceful. Her friends' parents made sure Emmy had a partner often enough that she had fun. Even so, the boys did not look forward to being her partner.

"Emmy, would you like to dance?" Stefan asked dutifully.

"Oh! That would be such fun!" Emmy squealed. Together they moved onto the dance floor, Stefan hoping to keep up with her, trying to anticipate when she would go this way and when she would go that way. He hoped the dancing ordeal wouldn't last too long. Luckily, there was never any question that Emmy had enjoyed it immensely, and her conversation was good enough to almost make up for her lack of grace. Stefan's mother beamed at him after the dance, thanking him for being such a good sport.

One evening after supper Emmy sat down beside her father on the couch. "Papa," she began, "you were saying last

week that Fritz and Alfred are starting to learn algebra. What is algebra?"

"Well, it is a way of generalizing arithmetic," he replied.

"What does that mean?" she asked.

"Well, you know that $9 + 7 = 16$, but that $7 + 9$ is also 16. With arithmetic you can say that clearly, but you can't make the more general statement that if you are adding two quantities you get the same answer regardless of which number comes first unless you use a lot of words. In algebra, we would say $x + y = y + x$ for all numbers x and y," her father explained.

"So algebra is like a code, letting you talk about numbers in general?" Emmy asked.

"That would be a good way to describe it," her father said. "I like to think of it as a kind of poetry. It allows you to express complicated thoughts using very few symbols, and yet it is clear to everyone who knows algebra. It is truly beautiful.

"Who invented algebra?" she asked.

"Hmm. It wouldn't be fair to say that just one person invented it. The way I think it happened is that one person got a good idea and tried it out, and then someone else came along and saw a way to take it further. That must have happened over and over countless times during many centuries," he explained.

"But who started it? Do you know, Papa?" Emmy asked.

"I guess the first person to formulate algebra as we know it was Diophantus, a Greek mathematician who lived in the third century AD," he said. "He is often called the father of algebra."

"What was he like?" Emmy asked.

"Actually, we know very little about him. Diophantine equations are named after him, but his papers have been lost.

Mainly, we know about him only from later mathematicians. But there is one tantalizing tidbit that we know, although it may or may not be true," he explained.

"Tell me! Tell me!" Emmy begged.

"All right. It's in the form of an algebraic puzzle. Your *Tante* Elisabeth might like it since she likes puzzles so much! I'll see if I can remember it: Diophantus's youth lasted one sixth of his life. He grew a beard after one twelfth more. After one seventh more of his life, he married. Five years later he had a son. The son lived exactly half as long as his father, and Diophantus died four years after his son. Do you think you could use that information to figure out how many years Diophantus lived, Emmy?" her father asked.

"I'll bet we'd have to start by adding $\frac{1}{6}+\frac{1}{12}+\frac{1}{7}$, because that would be the fraction of his life that Diophantus lived before he got married," Emmy said.

"Yes, that would be a good start, but in fact the easiest way to solve this is to use algebra. Shall I show you how it would work?" he asked.

"Yes, please, Papa. I want to learn this," Emmy said.

"All right. We don't know how many years he lived, so let's use the letter x to stand for the number that we don't know."

"So that's why we need the x! Would that mean that we would write $\frac{1}{6}\times x+\frac{1}{12}\times x+\frac{1}{7}\times x$?" Emmy asked.

"Yes, that would be a good beginning. Then what else would you add to that?" her father asked.

"We'd have to add 5 since we know that his son was born five years after his marriage. And the 5 doesn't need to be multiplied by x since it isn't a fraction of his life."

"That's right, Emmy, but what are you going to do about the next information: 'the son lived exactly half as long as his father'? How can you work that in?" her father prompted.

"Well, if the father lived x years, the son lived $\frac{1}{2}\times x$ years, and then the father lived another four years after his son died," Emmy said.

"That's good. So, can you write an expression for how long Diophantus lived?" he asked.

"Yes, I think you'd have to add up all these amounts: $\frac{1}{6}\times x+\frac{1}{12}\times x+\frac{1}{7}\times x+5+\frac{1}{2}\times x+4$, and that should be the total number of years that Diophantus lived," Emmy said.

"That's true, but don't you want to get a number? To find out the actual number of years that Diophantus lived?" he asked.

"Yes! Please show me how we can do that with algebra," Emmy asked.

"Think a minute. How many years did Diophantus live?" he asked.

"That's what we are trying to find out!" Emmy protested.

"Yes, but what have we been calling the number of years he lived?" he asked.

"We have been calling it x," Emmy said confidently.

"That's right. So we can write an equation: $\frac{1}{6}\times x+\frac{1}{12}\times x+\frac{1}{7}\times x+5+\frac{1}{2}\times x+4$ = the number of years Diophantus lived. What did we call that number of years?" her father asked.

"We called it x. So I think that must mean that we can write $\frac{1}{6}\times x+\frac{1}{12}\times x+\frac{1}{7}\times x+5+\frac{1}{2}\times x+4 = x$," Emmy said.

"Now I need to tell you a little more about algebra. The way we deal with equations," her father said, "is to always do the same thing to both sides of the equation. If we add 7 on

the right, we must add 7 on the left. If we multiply the left side of the equation by 3, we have to multiply the right side of the equation by 3. As long as we do the same thing to both sides of the equation, it is still true. We have to be careful to keep the balance. On this equation, you will need to start by adding up all those fractional parts of *x*, and …" her father's voice trailed off.

"I know! I know!" Emmy said excitedly. "I'll add up all those fractions. Just a second. I'll get it. Okay. Here it is: The fractions all add up to , so I can say $\frac{25}{28} \times x + 4 + 5 = x$. I know that 4 + 5 is 9, so that would be $\frac{25}{28} \times x + 9 = x$."

"Yes, Emmy, and here is where algebra can help you again. You can subtract $\frac{25}{28} \times x$ from both sides of the equation (remember we have to do the same thing to both sides of the equation), and that will give you this: $9 = \frac{3}{28}x$."

"Is that because you had 1 *x* on the right, so you subtracted $\frac{28}{28} - \frac{25}{28}$ to get $\frac{3}{28}$?" Emmy asked.

"That's right, her father said. "Then, since you want to know 1*x*, you use the reciprocal of $\frac{3}{28}$ (that would be $\frac{28}{3}$) and multiply both sides of the equation by it. When you multiply , $\frac{3}{28} \times \frac{28}{3}$what does it give you?" he asked.

"That gives me 1, of course. Then I could say $\frac{28}{3} \times \frac{9}{1} = \frac{28 \times 3}{1} = 84$. So Diophantus must have lived for 84 years. I like algebra!" Emmy announced.

"Yes, it is a very important part of mathematics," her father said.

"Papa, I'd really like to learn algebra," Emmy said. "Could you teach me?"

"I don't see why not," her father admitted. "I'll bring an algebra book home for you tomorrow," he promised.

Professor Gordan Comes for Supper

Later that week, Professor Dr. Paul Gordan, another mathematician at the university and a close family friend, joined them for supper.

When the conversation slowed down and *Frau* Noether noticed that Professor Gordan's plate was empty, she asked, "Won't you have a couple more *Bratwürste,* Professor Gordan?

"Yes, please, *Frau* Noether. They are excellent, I must say!" Professor Gordan said. "You know, in other parts of Germany the *Bratwürste* are big and fat—I love our small ones. They are so succulent and full of flavor, and you have prepared these perfectly! They are delicious."

"Papa," Emmy said as she finished her last *Bratwurst*, "I started working in the algebra book after school today. I'm trying to figure out how to work with positive and negative numbers. The book gives me some rules. Do I just have to memorize them?"

"Are you studying algebra?" Professor Gordan asked. "I hadn't realized it was part of the curriculum at the *Töchterschule*."

"It isn't," Emmy's father explained. "She was curious, and I couldn't see that it would do any harm."

"I think you are right," Professor Gordan said. "A bright child should be encouraged to exercise her brain, and I have

to confess that I'm curious about how you are going to answer her question."

"Alfred," Herr Noether said, turning to his oldest son, "have you learned to work with positive and negative numbers yet?"

"Yes, Papa. I know that when you add two positive numbers the result is positive, and when you add two negative numbers the result is negative. There are some other rules, but I don't remember them," Alfred admitted.

"That sounds like an answer that won't please Emmy!" her father said. "Remember, she doesn't want to simply memorize rules. Fritz, what about you? Can you answer Emmy's question?"

"Don't you get a positive number both when you multiply two positive numbers and when you multiply two negative numbers?" Fritz asked.

"Hmm. We seem to have a problem," Professor Noether said. "Both the *Gymnasium* students have memorized some rules for working with positive and negative numbers, but it seems to be rote memorization and nothing more. I don't like it any more than Emmy does. It sounds as if my sons have no concept of what negative numbers are or how you can manipulate them intelligently."

"I hate to interrupt the lesson," Emmy's mother said, "but I wonder if anyone wants any more *Bratwürste* or cheese? How about another slice of bread, Professor Gordan?"

"Thank you! This was just right," he answered.

"All right, then, Emmy, would you please wipe Robert's face and hands and clean up his place. Here's a washcloth. Alfred and Fritz, would you help me clear the table before we bring out the fruit compote for dessert?"

"Yes, mother," the children said.

After supper, the Noether family and Professor Gordan settled down in the living room to continue their discussion.

"Papa," Emmy asked, "isn't there a way to think these negative numbers through? Do I really just have to memorize a bunch of tricks?"

"It seems to me that it would be better to try a more thoughtful approach," her father agreed. "Why don't you try thinking it through? I think we all agree that when we add two positive numbers we get a positive answer. What about adding two negative numbers?"

"Alfred says that would be negative, but I don't like it. Why would it be negative?" Emmy asked.

"Let's say that Alfred has no money but he owes you three *Reichsmark*, and then he borrows four more *Reichsmark* from you so he can buy himself a bicycle. How much money does Alfred have then?" Professor Noether asked.

"Well, all he has is a debt! Poor Alfred!" Emmy said. "He actually owes me 3 + 4—that would be 7—so he would have 7 less than 0 *Reichsmark*. Would that be –7 *Reichsmark*?"

"That's right!" her father agreed. "So if he borrows two more *Reichsmark* from Professor Gordan for a bicycle pump, how many *Reichsmark* does Alfred have then?"

"Well, he would be in debt a total of 7 + 2 *Reischmark*, and that would be –9 *Reichsmark*," Emmy said.

"Now, what if I gave him 10 *Reichsmark*—and this time it is a gift, not a loan, so he wouldn't need to pay me back? If we add his 10 *Reichsmark* to the amount that he already had, how many *Reichsmark* would he have then?"

Emmy thought about it a minute. "He used to have 9 less than 0, so your first 9 *Reichsmark* should bring him up as far as 0—the middle ground, where he has no money and no debt. Then the last *Reichsmark* should bring him up to positive 1 *Reichsmark*," Emmy said cautiously. "Was that addition?"

"Yes it was! So now you know that (–3) + (–4) + (–2) + 10 = 1," her father said. "You seem to be able to add positive and negative numbers without a rule."

"Can we think it all through?" Emmy asked.

Paul Gordan (Courtesy of Stadtarchiv Erlangen)

"I would think we could," her father said. "Remember, somebody originally figured out all of this, and then somebody else came along later and wrote the rules."

"Thanks! That helps a lot," Emmy said. "I think I can learn algebra after all!"

"That was fun, Noether!" Professor Gordan said. "Let me know, Alfred, when you need the money for that pump! It seems to me that if children learn to think as they calculate, they will do it much more accurately, because it will mean something to them. I love to calculate, and it annoys me to see young people who are unable (or is it unwilling?) to calculate quickly and well. I suppose," he continued to Emmy's father, "that I am a much bigger fan of calculating than you are, Noether!"

"Yes, Gordan," *Herr* Noether said, "I think you get a lot more pleasure out of it than I do."

"But there is no doubt that calculating is the foundation of mathematics," Professor Gordan said.

"Gordan, I disagree!" *Herr* Noether said. "I think the overriding concepts are more important than the calculating."

"Yes, there's where we differ," said Professor Gordan. "I think the concepts are important because they allow us to understand the all-important calculating." Then, turning to Emmy, he said, "Emmy, I think you should keep on with your algebra! I'll be interested to see how you progress from here."

When she was 14 years old, Emmy's friend Anna Herder, who had been her classmate since the second grade, had been

absent from school for several weeks. Emmy was upset, and she asked her mother about it.

"Mother, why hasn't Anna come back to school yet? Is she still sick?"

"Yes, Emmy, I'm afraid she is," her mother said. "Anna's mother is worried. She has always been healthy, but she doesn't seem to be able to shake this illness."

"Can't the doctor do something for her? Can't he give her a pill?" Emmy asked.

"Apparently the pills haven't helped, Emmy. It is very sad," her mother admitted.

"What is going to happen to her?" Emmy asked.

"I don't know, dear. I don't know," her mother said quietly. "Remember her in your prayers."

A few weeks later when Anna died, Emmy was devastated. "Mother, Anna was my friend. I don't understand. Why did she have to die?"

"Emmy, I can't explain it," her mother said. "We'll probably never know. She was a sweet girl and a good friend, wasn't she?"

"Yes," Emmy said, "she was my best friend. I don't understand why she had to die."

"That is something we often don't understand, Emmy," her mother said. "We are born and most of us grow up to lead happy, productive lives. It must be very hard for a mother to watch her child die. Parents have great hopes for their children. That is why they work so hard at bringing them up well."

"What would you do if one of us died?" Emmy asked.

"That's hard to answer, Emmy," her mother answered. "Of course your father and I would be very unhappy. We must

remember to be particularly kind to Anna's parents, who have lost their only child."

Several months later, one evening after supper in that prosperous and secure time around 1897, Emmy snuggled up to her father on the couch once again. She was 15 years old. "Papa, you've told me about Diophantus. You said that he is the father of algebra. Who is its mother?"

"The mother of algebra?" he asked. "As far as I know it doesn't have a mother unless you want to call on Hypatia for that role."

"Who was Hypatia?" Emmy asked.

"Well, actually, her story is rather tragic," her father began. "She lived perhaps 100 years after Diophantus in the city of Alexandria on the Mediterranean coast of Egypt. Alexandria was the Göttingen of the ancient world—it was where scholars gathered. By all accounts Hypatia was a true intellectual whom everyone admired. Much of what we know of Diophantus's work comes to us through Hypatia's writing."

"She sounds very important, Papa," Emmy said. "Why is her story tragic?"

"Let me continue," her father said. "She was apparently a captivating lecturer who was able to bring mathematics to life. People came from far away to hear her lecture on a wide variety of topics, not just on mathematics, and the city leaders often consulted her when something important happened because she was so well informed on all subjects. One of her great interests in mathematics was conic sections."

"What are they?" Emmy asked.

"They are curves that you can find if you cut a cone at any of several angles," her father answered. "There are parabolas, ellipses, circles, and hyperbolas, and she did serious work on organizing those curves." Her father was sketching each of them as he talked. "We now have algebraic equations for each of them. For example, a circle with a radius of five can be described with the equation $x^2 + y^2 = 25$. You will learn about conic sections if you continue with algebra."

"Did Hypatia use *x*'s and *y*'s?" Emmy asked.

"No, but neither did Diophantus. Earlier algebra used only words, although sometimes those words were abbreviated. Variables such as x and y didn't appear in mathematics until more than a thousand years later—it was Descartes who set us up to use them in the 1600s. The sad part of Hypatia's story is at its end. At a time when the Christian church was rapidly expanding, Hypatia chose *not* to become a Christian. Because the Christians considered her a pagan for still believing in the Greek gods and goddesses like Hera and Zeus instead of their God, they attacked her. They may have feared her influence over the intellectuals in the city, as there is no doubt that the city leaders respected her and went to her for advice on important matters. For whatever reason, the mob murdered her cruelly. It's a horrible story."

"Could that happen to a mathematician today, Papa?" Emmy asked quietly. "Could they do something like that to you? We aren't Christians."

"No, Emmy, we aren't Christians," her father reassured her, "but the Judeo-Christian tradition is strong in Europe now. The Jewish and Christian cultures today are virtually the

same. A mathematician, whether he is Christian or Jewish, has nothing to fear in our modern world."

"That's good," Emmy said as she snuggled even closer against her father's shoulder.

A little more than 20 years later, in 1920, both Emmy and her father decided to join the Protestant church, the dominant church in that part of Germany. As non-practicing Jews, they felt thoroughly integrated into German society, and were by anyone's definition highly successful, professional academics. The family had supported Germany during World War I—Emmy's brother Fritz had served in the German army as a front-line combat soldier for four long years, earning the iron cross for bravery. In 1920, because the post-war Weimar Republic of Germany was a liberal, democratic, and modern state with religious tolerance for all, it was an appropriate time for the Noethers to make this move toward the mainstream. In 1920, no one could have predicted the appeal or success of the anti-Semitic Nazi regime only 13 years later.

Part II
Beyond the *Töchterschule*

A High School Diploma

When she was 15 years old, Emmy finished her studies at the *Töchterschule*. This did not include the *Abitur* (high school diploma) because she was "only" a girl. For the next three years, her parents hired tutors to help her with her English and French, making sure she mastered pronunciation and a more complete vocabulary in those languages. Although it was obvious to anyone who heard her speaking French or English that German was her native language, she could make herself understood, and her grammar was flawless.

In 1900, when Emmy was 18 years old, she passed the teachers' examination for the state of Bavaria, entitling her to teach English and French in a girls' school like the one she had attended. Her overall evaluation was "very good" rather than "excellent," because her presentation of the material had not been as smooth as the examiners expected. They found that although she had organized her sample lesson well, she had to start over several times when she got muddled as she was explaining a difficult concept. All her other scores were "excellent," however, and she was now ready to make her way in the world. For a homely but highly intelligent woman, this seemed like a good plan.

One evening after supper, Emmy and her parents sat down to talk about what she would do now. Her mother

thought a career in teaching was a good idea since it was beginning to look as if Emmy was unlikely to marry and raise a family. Her father was open to other ideas, but Emmy was prepared with her own plan. "I really don't want to teach foreign languages," Emmy explained. "I could do it. I have learned the languages, and my accent is pretty good. However, I think I would find it very dull very quickly. I know that when you teach children a foreign language you have to teach them a new way of thinking, and that can be interesting, but it would be the same challenge over and over, with every girl in every class, year after year. That strikes me as tedious. I don't think that's the way I want to live my life. I'm not sure I would do any better a job than *Fräulein* Jamin did with me. She certainly did not succeed in teaching me either French or English. She just supervised me as I memorized a lot of unrelated rules and words—Papa, could we use a mathematical concept and describe my mastery of French and English as nothing more than a collection of discrete (unconnected) facts?"

Ignoring the last question, her mother then asked impatiently, "So what are you thinking of doing? We could see about arranging a suitable marriage for you."

"I don't think so," Emmy replied. "I doubt that I have the patience to be a good wife and mother, and remember I still can't play the piano or sew very well! Actually, I have a completely different plan! I would like to get my *Abitur* and then perhaps go on to study at the university. Don't you think I could complete the *Abitur*, Papa? I think that I would like to study mathematics."

"Emmy," her mother said, "we are not joking here! You need to make realistic plans for your life. You are dreaming."

"Wait a minute, Ida," her father interrupted. "This may not be such a crazy idea. Emmy is bright, she has mastered algebra already, and if she is determined. I think she has demonstrated that she could study mathematics."

"In that case, we would have to hire more tutors to prepare her," her mother said grimly.

"Yes, but we can afford it. In addition, I think we could arrange for her to audit some classes at the university. Some professors would certainly allow her to observe their classes—at the very least she can sit in on my classes and Professor Gordan's. I believe she could get ready to take the *Abitur* examination at the *Real Gymnasium* in Nürnberg," her father said.

"I suppose it's possible," her mother admitted. "She might come out with an *Abitur* after all that, but she certainly couldn't go on and study formally at the university. That is utter nonsense. Emmy, I thought you were a practical child!"

"I think this might work, Ida," her father countered. "I believe Emmy could do the work, and, you know, there have been at least two women who earned doctoral degrees in mathematics from Göttingen. I don't see why Emmy couldn't follow in their footsteps."

"Outrageous!" her mother muttered under her breath.

Between 1900 and 1902, Emmy audited a variety of classes at the University of Erlangen. Her brothers Alfred and Fritz were also studying at the university, and their father was one of their favorite lecturers. Some days, the three younger Noethers and their father Professor Dr. Noether all participated in classes at the university. Emmy also worked with two

tutors, Dr. Christoph Schöner and Dr. Mäulwe, who prepared her in mathematics and other subjects. On July 14, 1903, she passed the prestigious *Abitur* examination at the *Gymnasium* in Nürnberg on her first try, giving her a document to prove that she was well educated. It was a major accomplishment for a woman. Her brothers Alfred and Fritz, who were younger than she, were able to move at a faster pace simply because they were young men, having gone straight through the local *Gymnasium* to earn the *Abitur*.

Emmy's parents still had to deal with their youngest child, Robert. After a few frustrating years at the *Grundschule*, he had started working in the small stationery store down the street,

Emmy and her three brothers (Courtesy of the archives of the Mathematisches Forschungsinstitut Oberwolfach)

but lately it had become clear that he couldn't continue. He would often mix up orders, sometimes delivering a package of paper instead of a requested account book, or misunderstanding how many pencils a customer wanted. His angry outbursts with unhappy customers in the shop had made him a serious liability for the business, and the owner had finally told Professor Noether that he could no longer employ the boy. Robert protested that the customers were mean to him, but his parents knew better. Robert was clearly not capable of doing the job, and he was also increasingly difficult to deal with at home. With time, it became clear that the only option was to put him in an institution where he could live a routine, scheduled life in a structured, supervised environment. His own family life was simply too complicated and too busy for him.

"Why can't Robert just live at home with us?" Emmy asked.

"We've tried that, Emmy," her mother explained. "He is a young man now, and he isn't happy with just staying at home. He wants to go out, but I can't always go with him, and I don't want him to get lost or someone to take advantage of him."

"Then I'll take care of him," Emmy proposed.

"No, Emmy," her father said. "Robert is too much for either you or your mother to handle. The institution we have found can care for him competently—that is what they do, and they do it well. He will be able to live his own life there. We'll see to it that he is comfortable and safe, but we simply can't keep him at home any longer."

The family always made sure to provide for Robert. After their parents died, Robert became Emmy's responsibility, and she provided for him until his death in 1928, although she was

forced to move him from the private institution that her parents had found into a state facility during the hyperinflation of the 1920s.

Studying at the University

With her *Abitur* in hand, Emmy was ready once again to broach the subject of her future with her parents. Her plan was to study at Göttingen, a university known since the time of Gauss for its brilliant mathematicians. As her father had observed, it had also been a little more accepting of women scholars than other German universities, although up to this time only Sophia Kovalevsky (a Russian) and Grace Chisholm Young (an Englishwoman) had been granted doctoral degrees in mathematics at Göttingen, and both of them had been foreigners. No German university had ever risked doing such an outrageous thing for a German woman. That could be allowed only for exotic foreigners, since they could be counted on to go back home again afterward. At that point, they would be somebody else's problem. The English and the French used the expression "blue stockings" for such women, and the term was always spoken with derision. Germany was not yet ready to face the difficult issue of home-grown, uppity women scholars. Everyone knew that the word *scholar* referred only to men.

Emmy knew the story of the Russian mathematician Sophia Kovalevsky, who had been granted a PhD at Göttingen in 1874, although Kovalevsky had never actually studied at Göttingen. She had done her doctoral work in Berlin through

independent study with Karl Weierstrass, Berlin's foremost mathematician, but the university there had refused to grant her the degree even though Weierstrass, with all his prestige, had pushed for it. No one questioned her intellect or her scholarship, but in Berlin rules were rules and, therefore, they had to be followed. Kovalevsky, who was grateful that Göttingen had been willing to accommodate her, went on to become a professor of mathematics in Stockholm—the first woman to be named professor in northern Europe—with help once more from her mentor Weierstrass as well as Gösta Mittag-Leffler, one of Weierstrass's former students.

Twenty-one years later, in 1895, British mathematician Grace Chisholm Young was awarded a PhD in mathematics at Göttingen after studying with professors there for three years, although not as a formally matriculated student—she wasn't a man, and universities had been established specifically to educate men. She had done her undergraduate work at Girton College, the women's college at Cambridge, England, and, although Girton did not award formal degrees to women, she had completed the program and performed brilliantly (if unofficially) on the university's mathematics examinations, outscoring most of the men who were officially competing. At Göttingen, she successfully defended her thesis in an oral examination, but even that was not easy for Young. The carriage that she had requested to take her to the university for her doctoral examination had departed without her when the driver had learned that only women lived in the house. He was no fool! He knew that only a man could take the PhD examination. Not to be defeated, she ran the distance as quickly as she could and arrived just in time for the examination. Even

though she was out of breath, she performed beautifully and was awarded the degree *magna cum laude*—with high honors. The stories of Kovalevsky and Young and their success in the face of challenges gave Emmy reason to hope for her own success.

In 1903, nine years after Young's PhD, Emmy registered to audit classes at Göttingen, attending lectures by Professor Schwarzschild in astronomy and by Professors Minkowski, Blumenthal, Klein, and Hilbert in mathematics. Since she wasn't shy, the professors were impressed by her penetrating questions and encouraged her to continue her studies.

Felix Klein (Courtesy of Staats- und Universitätsbibliothek Göttingen)

Felix Klein was the most prominent of Emmy's professors. He is known even among non-mathematicians for his invention of the Klein bottle which was one small part of his work in topology, the study of geometric figures in space as they are stretched and twisted but not torn. The Klein bottle, which has only one surface and curls back inside itself, yields a solid figure with only one surface (its inside is an extension of its outside). It is the three-dimensional equivalent of the Möbius strip. Although a Klein bottle is a challenge to make, you can easily make a Möbius strip by taking a long strip of paper, giving it a one-half twist, and taping the two ends together. Like the Klein bottle, it has only one surface: if you take a pencil and draw a line down the middle of one side, going on as far as you can, you will return to the beginning of your line without ever picking up your pencil.

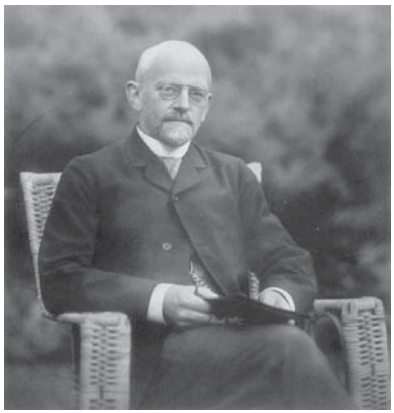
Klein had begun his career in 1872 at Erlangen, where Emmy would be born ten years later. While in Erlangen, Klein had delivered a speech outlining the "Erlangen Program," characterizing the various branches of geometry (Euclidean, hyperbolic, projective, topological) in terms of the groups of transformation that leave the properties of the geometrical objects invariant. At the time, this process involved intense calculations that often went on for 15 or 20 pages.

After one year in Erlangen, Klein moved to Munich and then on to Leipzig before settling down in 1886 in Göttingen, where he established the leading mathematical center in Europe. It was Klein who brought Hilbert from Königsberg to Göttingen, a move that guaranteed Göttingen's status as the mecca of mathematics of the time. By the time Emmy began her studies at Göttingen, Klein was no longer producing origi-

nal mathematics, but he continued to be a major force within the mathematical world for another 40 years, providing structure and encouragement for younger mathematicians.

As a young student, Emmy was most intrigued with the work of David Hilbert, whose 1899 book *The Foundations of Geometry* had brilliantly organized geometry into a complete axiomatic system, taking Euclid's work as his starting point and carrying the subject forward into a unified, consistent whole. Hilbert had begun the serious work of restructuring algebra in 1900, searching for an axiomatic basis for it in the same way that he had reformulated geometry. This was a revolutionary project, aiming to replace the straightforward solving of equations with an all-inclusive structure. His plan was that it should make very few assumptions (axioms), and build the rest of algebra on those axioms, following logical, rigorous steps to develop and prove first one and then the next and then the next theorem. It was this work in algebra that was to captivate Emmy, leading her to develop modern (or abstract) algebra into the coherent, strictly axiomatic approach for which she is famous.

Also in 1900, at the Second International Congress of Mathematicians in Paris, Hilbert had presented a list of the problems that he felt mathematicians needed to address during the twentieth century. Hilbert's goal was to establish mathematics in such a rigorous way that there would truly be no uncertainties. He was convinced that, given the proper tools and enough time, mathematicians could always prove that something either is or is not true within any clearly defined structure. This is sometimes called the "law of the excluded middle," meaning that there is always a dividing line between what is and what is



David Hilbert (Courtesy of Staats- und Universitätsbibliothek Göttingen)

not, and all possibilities are on either one side or the other of that line. Hilbert's list of 23 problems set the stage for mathematics in the twentieth century, in the same way that the Clay Institute's Millennium Prize Problems of 2000 challenged the mathematicians of the twenty-first century with seven difficult but as yet unsolved problems, only one of which (the Riemann hypothesis) remained from Hilbert's list in 1900.

After one semester at Göttingen, Emmy, who had become sick, returned home to Erlangen to recuperate and quietly study on her own. While she was recovering, the University of Erlangen made the decision to allow women to matriculate as regular students—German society was indeed becoming more modern. When she was feeling better, Emmy decided to remain in Erlangen and seriously pursue a university degree

there, living at home in the supportive atmosphere of her family, rather than return to Göttingen where she would still be limited to auditing classes. By this time, Emmy was a scholar with serious goals, and a doctoral degree was the necessary first step.

At Erlangen, Emmy studied under both her father and Paul Gordan, known as the "King of Invariants" because of his extensive research in this field. Gordan, who was seven years older than Emmy's father and a close family friend, became Emmy's *Doktorvater* (mentor), and she was his only PhD student. For the rest of her life she kept a picture of Gordan above her desk. She completed her PhD at Erlangen *summa cum laude*—with highest honors—in 1908, the first German woman to earn a doctorate in mathematics at Erlangen. Her dissertation explored algebraic invariants. It was highly technical, with many calculations following the style of Gordan, an algorithmist. To him, calculations were key. To Emmy, such calculations were mind-numbing work, but she was determined to succeed.

Gordan was famous for his walks that he took with a group of students, explaining in detail how to do a certain mathematical calculation. If no students were around, he would walk by himself, calculating in his head as he went, often completing a long calculation in such detail that when he returned home there was nothing left to do except write down the calculations from memory. With Gordan's work, there were never any cross-outs—the calculations appeared on the paper in perfect order.

If Emmy learned nothing else from Gordan, she learned his practice of walking while doing mathematics. She too was

a "peripatetic scholar"—walking and talking with students, occasionally stopping at a café for a refreshing drink but more often simply sitting down under a shady tree, all without interrupting the mathematical conversation. Emmy referred to this as "math talk." Her students, who sometimes worried that she would step out in front of a moving car as she pursued a mathematical concept, tried to look out for her. Mathematics was her consuming interest—really her only interest besides her students. Mathematical traffic was the only kind of traffic that could get her attention.

Gordan was not an abstract thinker. He produced concrete work, calculations that went on for pages and pages without any words at all. Mathematical symbols were his written language. In his classes and his writings, he rarely discussed big concepts, focusing instead on the details. He was determined to solve the theorem of invariants (often called the Gordan problem) by brute force.

After completing her degree, Emmy Noether worked in mathematics in Erlangen from 1908 to 1915 without a formal academic position and without pay. During this period, she began to explore some of the concepts she had learned from Hilbert in Göttingen, seriously pursuing the field of abstract algebra. Hilbert, who had met Gordan in 1888, had been intrigued by "Gordan's problem." Gordan had proven it for one class of invariants, but so far no one, including Gordan, had been able to make a more general proof. Hilbert considered the problem for several months and then announced that he had solved it. His solution, however, had no lengthy Gordan-style calculations. Instead, it treated the question by looking at groups of equations that behaved in predictable ways. Set

theory looks at the cardinality of a set—how many members the set has. Two sets that have the same cardinality are considered equivalent. Hilbert was pursuing a theorem of algebra that would eclipse the fundamental theorem of algebra, which had been proven by Gauss half a century earlier. Hilbert explored polynomials more broadly, and he succeeded in showing that a solution to Gordan's problem must exist, although he neither showed how to get that solution, nor guaranteed that it was unique. Gordan's reaction to Hilbert's proof was his famous comment: "That is not mathematics; that is theology," comparing it to the philosophical search for proof of the existence of God. Although Gordan later recognized that Hilbert's proof was indeed mathematics of a new and exciting variety, by that time, he was no longer able to do anything but observe the new field from outside.

Emmy Noether's new abstract algebra, which she pursued seriously after completing her degree, was entirely conceptual. She studied commutative rings—sets of numbers or concepts that one could combine according to specific rules and obtain a result that was also a member of the original set. She was particularly interested in the structure of those rings, and that structure allowed her to use set theory and symmetry to deal with algebraic objects rather than with specific equations. Her ability to elevate the specific to the abstract—to see the whole forest at a glance rather than focus on the individual trees—was astonishing. No one had ever gone this far in algebra before.

In later years, Noether dismissed her doctoral dissertation as garbage—"a jungle of formulas"—because it consisted mainly of pages and pages of calculations. She is often de-

scribed today as the mother of modern algebra, because she gave the new field the axiomatic structure that Hilbert had been seeking. She was able to organize the body of knowledge in a rigorous way without the use of algebraic calculations, placing her own new discoveries at the heart of the field. Gordan's main distinction today may be that he was Emmy Noether's doctoral professor and advisor.

Several years later, Karl Petri, who had known Noether when they were both students in Erlangen, asked her to look over a manuscript on a topic similar to her thesis. She declined, saying she had completely forgotten that part of mathematics, and recommended another reviewer. Another time when she boldly criticized a dissertation that had been praised by a well-respected professor, she was forcefully reminded that it had much in common with her own thesis! Although she had decided that that kind of mathematics, so heavy with calculation, truly was rubbish, it was not politic to speak out so strongly against the views of a lofty professor.

At this time in Erlangen, *Fräulein* Doctor Emmy Noether (as she would have been formally addressed) was living at home with her parents and brother Alfred, doing her mathematical research and occasionally helping her increasingly disabled father with his teaching. He was delighted to have her help him in this way—he knew that she was an excellent mathematician, and he was happy to acknowledge that fact. There was no doubt that she knew the material well, although he was aware that she was not a compelling lecturer. Her less-than-perfect performance on her teacher examination when she was completing the requirements to teach English and French was typical for her. Although she knew the material thoroughly,

her teaching was always flawed because she was much more interested in the development of the concepts than she was in their presentation. Once she understood a concept completely, the process of teaching it was nothing more than dull routine to her.

Emmy's brother Alfred, one year younger than Emmy, had studied chemistry at Erlangen and earned his PhD there in 1910, a year after Emmy completed hers. Alfred had chronic health problems, however, and remained at home after completing his degree, trying to keep up with his field at the same time that he was trying to regain his physical strength.

Her brother Fritz, who was two years younger than Emmy, studied mathematics and physics at Erlangen and Munich and completed his PhD in mathematics at Munich in 1909, the same year as Emmy. Fritz taught at the Technical University of Karlsruhe, and then, after serving for four years as a soldier during World War I, he accepted a call to be a professor of mathematics at the Technical University of Breslau. Fritz was an excellent applied mathematician. He also made one very important contribution to pure mathematics, the Noether Index of an operator. This is sometimes erroneously called the Fredholm Index, but Fredholm had nothing to do with it.

After Gordan retired, Emmy Noether began working closely with Ernst Fischer, Gordan's successor at Erlangen, and he rapidly became her new mentor. Fischer, who was seven or eight years older than Noether, was also interested in Hilbert's revolutionary approach to algebra, and he and Noether both loved engaging in "math talk" together.

Richard Dedekind (Courtesy of Staats- und Universitätsbibliothek Göttingen)

During one of their first meetings, Fischer asked her, "You have read Dedekind's work, haven't you?"

"Yes. When I returned to Erlangen after my semester at Göttingen, I was sick and had lots of time for reading," Noether said. "I asked my father to locate a copy of Dedekind's *Lectures on Number Theory*, and he checked it out of the university library for me. Hilbert had talked about Dedekind's work in the course I audited with him, and I decided to read it myself."

"Which edition was it?" Fischer asked.

"I don't know. Does it matter?" Noether asked.

"Actually, it does," Fischer explained. "Let me give you a little background on him. Dedekind studied at Göttingen with Gauss—he was actually Gauss's last doctoral student—and he worked closely with Dirichlet, another student of Gauss who

was Gauss's successor at Göttingen. Although Dedekind was a humble man, his work is monumental. He always said that his book *Lectures on Number Theory* was nothing more than an exhaustive study of Dirichlet's lectures at Göttingen, but it is far more. In each of the four editions, he painstakingly refined his ideas, and if you study all four editions you can see the development of his thinking. I don't think Dirichlet would recognize his own work in the last edition. It is pure Dedekind."

"And does the university library here have all four editions?" Noether asked.

"Yes, it does. If I were you, I would read the first edition carefully and then move methodically through the other three," Fischer said.

Ernst Fisher (Courtesy of Peter Roquette)

"You will not be surprised to hear that Gordan never mentioned Dedekind," Noether said.

"No," Fischer said, "that doesn't surprise me at all. Gordan is an excellent mathematician, but you are moving in directions he will never fathom, and which, I suspect, he wouldn't even like. Have you discussed your work with Gordan lately?"

"No," Noether said. "He doesn't seem to be particularly interested in what I am doing now."

They would talk in Fischer's office or in a café over a cup of coffee, exploring the abstract algebra that fascinated them both. Often they discussed Dedekind's ideas as she studied the first edition of *Lectures on Number Theory* and then each of the others, examining how Dedekind developed his ideas from one edition to the next. Sometimes Noether and Fischer would attend a mathematics seminar and then walk around the old town wall together and continue to talk. Their conversation was lively, as they enthusiastically developed new ideas, building on one another's creativity and knowledge. After these meetings, Noether would continue to mull over the concepts, and as soon as she got home, she often wrote a postcard to Fischer as a continuation of their discussion. Reading Noether's postcards is like listening to one side of a spirited discussion of abstract mathematical concepts. Fischer, who saved them all—hundreds of postcards dating from 1911 to 1929—knew that they were the work of an important new mathematician. He was repeatedly stunned by her genius, and he quickly realized that Noether was more brilliant than anyone else he had ever met. Their professional relationship clearly delighted them both.

In 1908, as a promising new mathematician, Noether was invited to join the Circolo Mathematico di Palermo—the Mathematical Association of Palermo, Italy—and the Deutsche Mathematiker-Vereinigung—the German Mathematical Association, commonly called the DMV. She attended a DMV meeting for the first time with her father in Rome in 1908. At the DMV meeting in Salzburg the following year, Emmy Noether was the only woman who presented a paper. Since the DMV provided a congenial setting for young mathematicians to present their research and to communicate with other mathematicians, Noether particularly enjoyed these opportunities for "math talk" with both young scholars and well-established professors. She continued to enjoy such meetings for the rest of her life.

In 1910, Hans Falckenberg, who had played with Emmy and her brothers when they were children, completed his dissertation in mathematics at Erlangen under Emmy's father. Although Falckenberg had planned at first to study law, he soon changed to his real love, mathematics. His thesis focused on a topic that both he and Emmy had been working on, but when she realized that he had made such a good start on it, she urged him to pursue it seriously, telling him that she would be delighted to help him as he developed the topic while she would go on with another idea. She often referred to Hans as her first doctoral student, although, at least on paper, Max Noether was his *Doktorvater*. When his dissertation was completed in 1911, Falckenberg officially acknowledged Emmy Noether's help on the entire project. After serving with Emmy's brother Fritz in the army during the Great War, he later became a professor of mathematics at the University of Giessen in Germany.

Part III
The Young Scholar

Emmy Moves to Göttingen

In 1913, on the eve of World War I, Emmy Noether again presented a paper at the DMV meeting, this time in Vienna, and once again she enjoyed exploring mathematics with her colleagues from throughout the German-speaking world. After the meeting she visited Franz Mertens, the *Doktorvater* of her friend and mentor Ernst Fischer, at his home in Vienna so they could discuss her research. Mertens, who knew of Noether's collaboration with his former student Fischer, was a senior mathematician who had written a completely original and valid proof of Gordan's theorem of invariants. He, like Hilbert, had not attempted in his proof to provide a method for finding the invariants—he simply established that the invariants existed. Gordan would never have been satisfied until he found all of them. It turned out that Noether's novel approach to algebra was much closer to Mertens's work than to her *Doktorvater* Gordan's.

Many years later, Mertens's grandson remembered Noether's visit. According to him, she looked something like a Catholic chaplain, dressed in a long shapeless coat and a man's hat, with short hair (unheard of for a woman at that time) and a shoulder bag that hung diagonally from her shoulder

to her waist. Her shoulder bag reminded him of a train conductor's bag of tools for selling and punching tickets. To a small child, Noether was an enigma. In Mertens's grandson's experience, women were mothers, and Noether had chosen a totally different role for her life. Although she was neither wild nor offensive, she was certainly odd, and her appearance was somewhat masculine. When he asked his grandfather who it was (not knowing whether to use the pronoun he or she), his grandfather explained that Noether was a great mathematician and a wonderful woman with whom he was discussing important work.

Back home in Erlangen, Emmy, Alfred, and their parents enjoyed congenial evenings in the family's apartment. Some evenings all four of them simply read books or newspapers quietly, some evenings Emmy's mother played the piano while the other three read, but some evenings they had lively discussions on a wide variety of topics ranging from politics to chemistry to mathematics. Her mother no longer pushed Emmy to practice the piano—even she had accepted Emmy as a professional mathematician—and they were all proud of her.

One evening Emmy's parents were having a quiet talk before the young people returned home for supper. "Max," Emmy's mother said, "tell me about this young professor Fischer."

"Oh, well, he's a brilliant mathematician. He and Emmy seem to be working together beautifully," Max replied.

"That's what I thought," her mother continued. "Do you suppose we should invite Professor Fischer over for dinner one Sunday? Doesn't he live alone? Emmy isn't getting any

younger, you know, and she certainly will never be able to forge a relationship with someone outside her field. It looks to me as if he might be the perfect husband for her."

"I don't know, Ida," Max said. "I'm not sure that Emmy has any interest in that kind of relationship. She likes to dance, but I don't think she has any interest in a romantic relationship with a man. I could be wrong, but I don't think so."

"She does seem to like him a lot," her mother continued.

"Yes, she respects him greatly as a mathematician," Max said, "and so do I. He is helping Emmy explore her field in very interesting ways, and the two of them seem to have a vibrant friendship, but it looks purely professional to me."

"Would you please sound him out and see if he would like to come for dinner next Sunday?" her mother asked.

"Don't you think we should talk with Emmy first?" Max asked.

"No, she would never agree," her mother said. "Let's just ask him."

Emmy, however, quickly figured out what her parents were plotting and she made it clear that she was unwilling to join in. Her mother was a strong woman, but it turned out that Emmy was even stronger. The matchmaker never had a chance.

"Emmy," her father said one evening, "I believe we should make a trip to Göttingen for you to spend a week or two working with Professors Klein and Hilbert. You have done major work in Hilbert's new algebra, and I think it is time for you to collaborate actively. He is the most important scholar in your field, and you are part of the new generation that will be car-

rying that field into the future. Why don't I write to Professor Klein and see what he thinks?"

Emmy was delighted with her father's suggestion. If she had tried to make such a trip alone, it would have been awkward, at least at first. Since her father and Klein had worked together many years earlier, her father could open things up for her. In fact, her father was as excited about visiting Göttingen as Emmy was.

A week later, Max Noether received an enthusiastic letter from Klein, begging them to come and welcoming them both to the mathematical world of Göttingen. Several days later, they boarded the train for the four-hour trip to Göttingen. While Emmy worked on some papers in their compartment on the train, Max happily sat and contemplated their visit to the mecca of mathematics. A mathematics student met them at the station and helped them take their things to a hotel before they went to have supper at the Hilberts' home.

"Professor Hilbert," Emmy began, "what fun it is to be back in Göttingen! When we first stepped off the train, I could already feel how vibrant and magical the town is. Do you ever get used to it?"

"It is a wonderful place to live," David Hilbert agreed, "but I guess we do sometimes get complacent." Turning to Emmy's father, Hilbert said, "Professor Noether, how nice to welcome you to Göttingen, too. I would like to introduce both of you to my wife Käthe."

"I'm pleased to meet you both," Emmy's father said.

"*Herr* Professor Noether, *Fräulein* Doctor Noether," Käthe Hilbert said, shaking hands warmly with both of them. "How wonderful to meet you both! May I offer you a glass of sherry?

You know that is the only thing that mathematicians at Göttingen would ever think of drinking before supper, don't you? Always the obligatory sherry."

"That would be wonderful, *Frau* Hilbert!" Emmy's father said. "It is so kind of you to invite us into your home."

"*Fräulein* Doctor?"

"Oh yes, please," Emmy heartily agreed. "I love a good glass of sherry!"

"I hope the journey wasn't too tiring," *Frau* Hilbert said.

"I found that I was very impatient to get here," Emmy admitted, "but actually the ride wasn't too long, and I was able to work on the train."

"I'm so pleased. David thinks that you are doing some important new work, *Fräulein* Doctor," *Frau* Hilbert said.

"I find it exciting at the very least," Emmy said.

The two weeks in Göttingen passed quickly as Emmy Noether got to know the two most important mathematicians in Germany. As Hilbert worked with her, he was astounded at her ability to manipulate abstract concepts in novel ways and at the speed with which she did it. He found that she had already taken algebra in directions that he had never imagined. Keeping up with her as she talked was a challenge in itself. In 20 minutes she was able to propose three or four radical new ideas that she wanted to explore. Hilbert was amazed. He could see that she was a phenomenon.

By 1914, Emmy Noether had published her first paper on the new algebra, "Fields and Systems of Rational Functions." In that paper, she pulled together the work that had been done in ring theory by such luminaries as Dedekind (who had begun the process of structuring algebra through axioms) and

Hilbert. As she incorporated her own discoveries into this new field, she presented her abstract algebra to the mathematical world. Much of the basic investigation for it appears in her extensive postcard correspondence with Fischer. When Fischer described some of her investigations in the *Göttinger Nachrichten*—the journal of the Scientific Society of Göttingen—with references to her paper, he brought attention to her as an emerging scholar in the field of invariants and abstract algebra. Although the correspondence paused for several years while Fischer served as a soldier in the German army in World War I, their fascinating correspondence resumed after the war. Throughout her life, Noether expressed her debt to Ernst Fischer for getting her started on abstract algebra, the field that she was destined to dominate for the next 20 years.

In 1915, Hilbert and Klein invited Emmy Noether to return to Göttingen—her dream ever since she had been forced by illness to leave Göttingen so many years earlier—and she was ready to work in earnest with her peers there. She hoped eventually to get an official position as lecturer or researcher at Göttingen, but that would not be possible until several more years had passed. For one thing, Germany had become severely impoverished by the war. However, even if there had been money to hire her at the university, what would soldiers returning home from the war think if they found a woman professor at that famous university? How would they react if they arrived at Göttingen and were expected to learn at the feet of a woman? It didn't impress the Germans that Sophia Kovalevsky had become a professor in Stockholm in 1883—that was just those

crazy Swedes! Furthermore, Kovalevsky had been an attractive young woman who had possibly charmed her way into the university, while Emmy had never been considered beautiful. In contrast, Germans understood what made a true scholar and Emmy failed that test on all points.

Emmy's mother decided that Emmy needed some decent clothes before moving to Göttingen. She summoned a dressmaker, who arrived with swatches of cozy wool in dark colors, fine cotton and linen for the removable collars, lovely silk for one fancier dress, and sketches of several different styles of dresses. Emmy and her mother chose several styles and fabrics before the dressmaker took out her tape measure and began to take detailed notes. A week later, the dressmaker returned with one partially completed dress to check the fit so that she could finish the rest of the outfits.

Frau Noether asked, "*Frau* Teubner, would it be possible to make this dress fit a bit closer at the waist? It seems rather shapeless on Emmy."

"No, Mother," Emmy interrupted. "I have to be able to breathe! The fabric is beautiful and *Frau* Teubner's work is skillful, so people won't be looking at me. I am a scientist, not a fashion plate. The mathematicians in Göttingen wouldn't recognize me in a stylish dress. As long as I look neat and clean, I don't think anyone cares what I wear."

In fact, there were people in Göttingen who criticized Noether's wardrobe, noting, for example, that she wore the same dress for weeks on end, but Noether was deaf to such comments. Clothes were a senseless nuisance to her, and the University of Göttingen never paid her enough to make it easy for her to keep her wardrobe up-to-date, even if she had wanted to.

No one ever suggested that Noether was either charming or beautiful. People said that she looked like a fat, plain, loud washerwoman in baggy clothes and thick glasses. That description, however, completely missed the essence of this remarkable woman: She exuded warmth and she was a brilliant but kind and gentle soul. Hermann Weyl described her as "warm, like a loaf of bread."

David Hilbert and Felix Klein in Göttingen were not put off by Noether's appearance. Hilbert, even with his well-known penchant for pretty girls, especially admired and respected this less-than-pretty, eccentric, and engaging woman. Physical beauty was one thing; the beauty of mathematics was quite another, and there was no doubt that Noether's mathematics was a thing of exquisite beauty. Hilbert and Klein were delighted to work closely with her. They saw that she possessed the vibrant and perceptive mind of a true scholar.

In 1915, Klein and Hilbert were working closely with Albert Einstein on his general theory of relativity, and they could see that Noether's expertise in abstract algebra would be pivotal in the field of relativity. Unfortunately, just two weeks after she arrived in Göttingen, Noether received word that her mother had died.

She immediately made her way to Hilbert's home to tell him her sad news. "Professor Hilbert," she began, "I'm very sorry. I have just gotten word that my mother has died. It was totally unexpected. I am afraid I must go home for a few weeks to settle things there before I can start my work here."

"*Fräulein* Dr. Noether, I'm so sorry!" Hilbert said. "Of course you need to go. Take as much time as you need."

Emmy's mother had recently had surgery on one of her eyes, and although she had seemed to be making a good recovery, suddenly she had died. Emmy and her mother had always been on good terms, but Emmy and her father were particularly close. When Emmy returned home to Erlangen, her main goal was to help her father and to make arrangements to care for him and her brother Alfred, whose health was still poor.

"Papa," Emmy told him the day after the funeral, "I believe I have found a woman who will be able to come to help you and Alfred every day and take care of your needs. She'll arrive in time to set out breakfast, she'll keep the house in order, and she'll prepare dinner for you every afternoon. I've asked her to come talk to us this evening. I hope you'll like her." When the woman came, they were all satisfied that she was competent, intelligent, and pleasant. Emmy was pleased. Within a few weeks, Emmy was able to return to Göttingen so that she could start on her career.

As Emmy Noether was establishing herself in Göttingen, Hilbert and Klein tried to arrange for her *Habilitation*, the formal recognition that she had moved a major step beyond the PhD, which would allow her to become an official member of the faculty. Although her publications were impressive by any standard, her *Habilitation* was not yet to happen. By this time, she had done some excellent research and had published several significant papers—it would have been more than enough for a man—but she was "just" a woman. *Habilitation* required a vote of the entire faculty, including philosophers and historians, as well as natural scientists and mathematicians. "How," they asked, "could we allow a woman to become a lecturer?

Next she will want to become a professor! And then what is to keep her from becoming a member of the faculty senate?"

In response, Hilbert contended, "Gentlemen, the position in question is at a university, not at a bathing establishment. Surely the sex of the applicant should not be a factor."

"Are you implying that this is some kind of farce?" they asked.

"No," Hilbert responded. "I assure you that I am totally serious."

Hilbert's solution to the problem was ingenious: Noether was at the university, and quite naturally she would be working with him. What he would do was advertise the classes under his own name, with the added information that he would have help from *Fräulein* Doctor Emmy Noether. In fact, Noether taught the courses entirely on her own, and there was only one real disadvantage for her: she was not paid for her work! Fortunately, Emmy Noether had had two uncles, her mother's brothers, who had left her a modest amount of money when they died. Her uncle Paul Kaufmann had been a successful businessman, and his brother Wilhelm had been a professor of international law at the University of Berlin. The money they had left to Emmy had allowed her to support herself as an unpaid scholar at Erlangen for several years, and now it would take care of her at Göttingen. Since her needs were never great, she didn't worry. She was just grateful that Hilbert and Klein respected her and treated her as a colleague. What she wanted was to talk and do mathematics, and that did not require a title or a salary.

"Well, Klein, I wonder what more we could have done," David Hilbert said to his colleague Felix Klein. "She's bril-

liant. We know she's brilliant. It's criminal that she still doesn't have an official position on the faculty. She may be the most impressive scholar we have."

"That's true, Hilbert," Klein answered, "but I guess the time is not right. You have to admit that she has a few strikes against her, the first being that she is a woman."

"That's true," Hilbert said, "but I doubt that it's the only factor. Do you suppose they are bothered by the fact that she is a Jew?"

"Many mathematicians are Jewish," Klein answered, "in Göttingen as well as in the rest of Germany. Furthermore, many of those other Jewish scholars have earned the right to be habilitated! Why not her?"

"You're right," Hilbert said, "but I think her Judaism was at least somewhere in the back of their minds. Beyond that, they are undoubtedly aware of her pacifism. She has been outspoken in her opinions on the war—she thinks it's stupid, pure and simple."

"Well, so do you, if I'm not mistaken!" Klein said. "And so do I! And we are both habilitated!"

"Yes," Hilbert continued, "but I'm sure there are some professors who have heard about her fascination with Russian philosophy—she seems to be most impressed with Marxism, and, as we both know, she never keeps her opinions to herself."

"I suppose you are right about that," Klein said. "Maybe it would be easier for her if she were a little less forceful in her conversations. And you have to admit that no one would ever describe her as winsome! She is certainly not another Sophia Kovalevsky!"

"But if they are excluding her because she is a woman, you would think it would be to her advantage not to be particularly feminine," Hilbert said.

"These arguments are never going to be truly rational," Klein said. "Perhaps it would be better if she were a little quieter—she loves a heated debate, and she doesn't lose ground graciously. Her booming voice has drowned out more than one adversary in debate, and she doesn't care who her adversary is!"

"But that's because she's right!" Hilbert said. "She is so much smarter than the rest of us that quite naturally we can't begin to keep up with her. She has to be forceful if she is going to show us where we are wrong."

"Perhaps what our colleagues are trying to tell us," Klein said, "is that they don't want to deal with this brilliant woman as a peer. She is not a person that the academy wants to have as its representative in modern Germany."

"But isn't the academy best represented by its finest scholars?" Hilbert argued.

"Yes, Hilbert, but sometimes those scholars make the establishment uncomfortable," Klein said. "I don't think we should give up, however. She appears to be willing to continue working with us for now without formal approval and without pay, and at the rate she is going, it will soon be impossible for the authorities to turn her down. She is simply too important."

"I hope it doesn't take too much longer, Klein," Hilbert said. "Her patience may have its limits, and what if somebody else figures out what a gem we have? Then we would really lose. We need her for our collaboration, possibly more than she needs us."

"I don't think we are in danger of losing her," Klein said. "Göttingen is the best place for her and she knows it."

When Emmy Noether first moved to Göttingen, she rented an attic apartment on Stegemühlenweg, a quiet street within easy walking distance of the university. The rooms were cramped and dark—it was truly a garret, always cold in the winter and sometimes oppressively hot in the summer. A simple gas burner and a sink served as her primitive kitchen for the many years that she lived there, but since a cup of tea and a bowl of pudding were her most ambitious cooking projects, the "kitchen" was adequate for her needs. Her students agreed that Noether's pudding, prepared and served after a Sunday afternoon walk, was divine! As they continued their

House on Stegemühlenweg 57 (Photograph by the author)

Dr. Oetker pudding mix (Courtesy of Dr. August Oetker Nahrungsmittel KG)

"math talk," Noether would open several packages of Dr. Oetker pudding mix, throw in some sugar and milk, cook it on the burner while occasionally stirring it, and serve it in her miscellaneous collection of cups and bowls, trying not to disturb the bottom of the pot where inevitably some of the pudding had burned. Her students loved the *Kinderfeste*—the children's hour—at home with their "mother" Noether.

When the maid washed up the dishes in the morning, she dealt with the hardened pudding stoically. *Fräulein* Doctor was odd but pleasant, and the bowls were always stacked neatly in the sink. After soaking for an hour, even the cooking pot usually came clean without too much difficulty. For many years Emmy lived happily in that attic apartment, which she jokingly called her "mansard" dwelling, the most comfortable home she would ever know in Göttingen.

Noether ate dinner every day at a simple restaurant near her apartment.

"Hello, *Fräulein* Doctor," the waitress greeted her every afternoon. "Would you like your usual beans and sauerkraut and boiled potatoes with a couple of slices of nice fresh bread and butter?"

"Yes, Liese," Noether responded. "It seems like the perfect day for that sort of dinner." Noether always sat at the same table and always ate the same food. Food itself was a trivial matter to her. If the food tasted good enough not to distract her from her mathematics, she gave no further thought to it.

A Lecturer at the University

By 1915, Noether had already become known in the field of abstract algebra. By 1920, she was considered a serious mathematician in her own right, not just an appendage to Hilbert and Klein. Building abstract algebra into a serious discipline, Noether began with the premise that algebra deals with sets of objects; some of those objects are numbers, some are polynomials, and some are abstractions like matrices. Her algebra was structured on rings and ideals, building on Dedekind's rings. A ring is a set of numbers or objects that is closed under addition, subtraction, and multiplication, always producing a result that remains within the original set of numbers or objects. In a ring, there must always exist an identity function (which produces no change in the quantity entered, such as adding 0 or multiplying by 1) and an inverse function (which returns a given quantity to the identity element, such as adding a quantity to its opposite). Using set theory, Noether was able to find a general solution to a given problem. Her solution was not dependent on the specific equation, involved no tedious calculating, and allowed her to describe the result to any similar problem as well. At first, abstract algebra was limited to commutative operations with real numbers, but soon she was able to use the same approach on problems involving

irrational numbers (numbers that cannot be expressed as the ratio of two integers or as terminating or repeating decimals), complex numbers (numbers with a real part and an imaginary part involving the square root of −1), and beyond. Dedekind, Hilbert and others had worked seriously in this field, but it was Emmy Noether who revealed the structure necessary to make it a united whole. Klein wrote in a letter to Hilbert that she was helping him greatly in his work in this field, and that he had great respect for her as a scholar.

As Noether taught classes on abstract algebra, she had to begin by describing the topic at an elementary level, urging her students to think conceptually with her. "Let's talk about the integers as a ring. Yes, integers are a ring, because you can add, subtract, and multiply them and you still end up in the set of integers. Also, the set of integers has an identity function (adding 0 or multiplying by 1) and an additive inverse function (adding the opposite). Now, because the set of integers is a ring, we can be sure that it will behave as rings always do."

"Doctor Noether, isn't division involved in rings also?" one bold student asked.

"No, no! You are talking about fields, not rings. We cannot include division in our definition of a ring! Think of what happens if we divide the number 24 by a perfectly innocuous number like 7. Do we end up within the set of integers? Of course we don't! Division would be very inconvenient, and for that reason it is excluded from rings. We include division in fields—the set of rational numbers is a field—but in fact the ring of integers is far more interesting."

"Is there another set of numbers that would qualify as a ring?" another student asked.

"Of course. Look it up in Dedekind!" she replied. "Now, we need to move on to the ascending chain condition for ideals."

"Have we dealt with ideals?" asked a student who was struggling to find his way in Noether's class."

The response was a chorus from the class: "Yes, we have! It is a subset of a ring—it is a very special ring."

Another student continued, "The set of even integers could be called an ideal of the ring of integers because if you add even integers together, you are sure to remain within the set of even integers. When you multiply even integers, you are guaranteed to remain within the set of even integers. That makes the set of even integers an ideal of the ring of integers. What you need to do is study Dedekind! It's all there!"

"That seems to take care of ideals," Noether said. "Are there any other questions before we go on?"

In 1916, Fritz Seidelmann, an instructor at a teachers college for women, approached Emmy Noether's father, Max Noether, in Erlangen, for advice on a paper he had written. "Do you think, *Herr* Professor," he asked, "that my paper could be developed into a dissertation? I think I would like to complete the PhD in mathematics if you think it is possible."

Looking over the paper carefully, Max Noether found it both interesting and original. He realized, however, that the paper was in Emmy's field, not his. There was no doubt in Max Noether's mind that it was important and good. "Yes, *Herr* Seidelmann," he replied. "I think this has real potential, but you should realize that the topic is much closer to my daugh-

ter Emmy's work using group theory with invariant equations than it is to mine. Professor Fischer could have worked with you, but, as you know, he is serving in the army. Why don't you write to my daughter in Göttingen? She seems to be recognized as the leader in this field, and I believe that she would be delighted to work with you. Here, let me give you her address: *Fräulein* Doctor Emmy Noether, Stegemühlenweg 57, Göttingen. Please write to her; send her your paper and ask her what she thinks. If she agrees, you would officially be my student, of course, but in fact my daughter would be your real mentor. I think that is what will be best for you."

Seidelmann did as Max Noether suggested, and Emmy readily agreed to the arrangement. She and Seidelmann exchanged many letters as he developed his ideas further. When Emmy returned to Erlangen between semesters, she enjoyed working with him one-on-one. When it snowed, however, there were some distractions. When she found her sled in the cellar of the family apartment building, she couldn't resist taking it out for a run down a nearby hill. Dressed in a long dark sporting costume made of dense gray wool and a cozy warm hat, she pulled the sled to the top of the hill, sat down on it, and off she went! Her happy face beamed from the folds of the scarf that was wrapped around her head and neck. As the sled bumped down the hill, she shrieked with joy. When she reached the bottom, she shouted to her friends, "That was a good run! Don't you love it? I say let's try it one more time!"

When Seidelmann finished his doctorate, the great mathematician Emmy Noether was immensely flattered that he dedicated his dissertation to her. When he passed the oral examination *summa cum laude*, it was hard to tell who was more

pleased, he or *Fräulein* Doctor Noether. Afterward, she always referred to Seidelmann as her second doctoral student.

In 1915, it had become clear that while people were going hungry, pigs were consuming vast quantities of basic food like turnips and cabbage and potatoes—food that could be used for nourishing people. Since pork was a miniscule part of the wartime diet, the mass slaughter of nine million pigs was ordered, producing two short-term benefits: pork for dinner (as well as sausages and other meat by-products) and more turnips, cabbage, and potatoes available for people to eat. The feast didn't last long, however. The winter of 1916–1917 in wartime Germany was known as the "turnip winter." Food was scarce throughout Germany, and most people subsisted on a grim diet of turnips relieved by the occasional cabbage or potato. Emmy, who was always a pacifist and never understood why the Germans had gone to war in the first place, continued to be irritated by the extreme and seemingly needless privations of the later war years. With her simple needs, she could easily eat turnips for her midday meal every day instead of her usual potatoes, beans and sauerkraut, but she knew that others valued a variety of foods, and even she had to admit that she was sometimes hungry. When her brother Alfred, who was exempt from military service because of chronic illness, died in 1918, she blamed his death on the war. Who could tell what great things he might have accomplished as a chemist if he had had an adequate diet during those hungry years? By war's end, everyone was hungry, cold, and tired, and Alfred apparently did not have the resilience to survive.

As she had done three years earlier for her mother, she made the sad trip home to Erlangen to bury her brother Alfred. Poor Alfred had been sick for a long time.

"Papa," she said as she embraced her weary father after her short walk from the station, "I'm so sorry you had to be alone with Alfred through his last illness."

"Yes, Emmy, he suffered," her father admitted. "But he never complained. And do you know that he never lost his sense of humor, even at the very end?"

"I'm not surprised to hear that, Papa," Emmy said.

"Just the other night he was talking about you—about how proud he was of you," her father continued. "He said that he always knew that you were a genius, and that he was so pleased by all that you have been able to accomplish. He predicted that you would go on to do very great things—he suspected that sometime in the future people would describe you as a great mathematician."

"He was always a dear boy. We will all miss him." Emmy quickly brushed away the tears that had welled up in her eyes and kissed her father on the cheek.

She hated to leave her father alone after the funeral, but he insisted. "No, Emmy, you must go back to Göttingen. You are a great mathematician, and Göttingen is the best place for you to make your career. I'll be fine here. Write to me when you get a chance, and come visit during your holidays, but you must persevere. I have friends who look in on me, I have a great deal of interesting reading to do, and the woman you found to help around the house takes care of me beautifully. Go back to Göttingen!"

In addition to physical hunger, German scholars felt isolated from the rest of the academic world during the war years. Hilbert would have been glad to collaborate with Whitehead or Russell in England, but that was not possible. German scholars were totally isolated and were limited to collaboration only within the German-speaking world. For mathematicians in Göttingen, working with Noether was at least some consolation.

As Noether worked with Hilbert and Klein in their collaboration with Einstein in 1917, they were all focusing more closely on one key point: the connection between invariants and Einstein's theory of relativity—although time was warped at great speeds, there were things that remained invariant. In a letter to Hilbert, Einstein wrote "Yesterday I had an interesting conversation with *Fräulein* Doctor Noether concerning her work with invariants. It struck me that you could look at this thing [relativity] from several different angles… It seems to me that [she] understands her work well." Tying differential invariants to linear algebra, Noether was able to provide the mathematical formulation for relativity, a chore that no one, including Einstein, could complete without her abstract algebra. As a physicist, Einstein knew that a mathematical proof was necessary, but he was not able to do it himself. Hilbert once commented that physics is much too difficult for physicists to do themselves! Noether's revolutionary techniques allowed her to approach Einstein's theory in a new and effective way and to do the "calculations" at the only possible level—that of abstract algebra.

In May 1919, with the war over and a shaky democratic structure in place in Germany, Noether made her second attempt at *Habilitation*. The scholars at the university decided to make an exception to their general rule. This time the University of Göttingen had become aware that Frankfurt was likely to offer Noether *Habilitation* at its university, and Göttingen certainly could not allow one of its most prominent scholars to be snatched away. With her application for the *Habilitation*, Noether submitted a *curriculum vitae*—a listing of her academic accomplishments—enumerating her published works, beginning with her dissertation. She went on to list the papers she had written while she worked first with Fischer and then with Hilbert. Then she described her unpublished work on the mathematical foundations of Einstein's general theory of relativity. This work is now referred to in physics as Noether's theorem.

On June 4, 1919, she was officially "habilitated," with her lecture on that theorem. She was pleased that she was finally respected as a scholar, although even then her position was tenuous. She would be allowed to lecture as a *Privatdozent* (private lecturer) in her own name, but still she would not be paid for her work. In other words, she had gained nothing more than the privilege of teaching. She didn't even have the authority to mentor doctoral students. As always, she did not complain. She had enough to eat, and she could pursue her mathematics in a congenial setting. The occasions when she did complain were not when she perceived that she had been treated badly—she complained only when she felt that someone else had been treated badly. Somehow she was able to proceed with her own work, seemingly oblivious to any injustices in her own situation.

Noether's accomplishments were truly impressive. Dedekind, Fischer, and Hilbert had set the stage for her work in algebra, but she was the genius who put it together into a unified whole, producing an axiomatic method for dealing with algebraic problems as structures with certain characteristics rather than as individual problems to be solved by tedious calculations. She was able to construct concise axioms for the groups under consideration and then use those axioms to prove theorems that led to a consistent algebraic theory. She had no interest in jungles of formulas, which she had not touched since the completion of her own dissertation. She did not need a specific example in order to develop her conceptual framework. Her mind thrived on abstractions—processing them so rapidly that it was difficult for others to keep up. By now the mathematical world well beyond Göttingen revered her as a genius.

For many years, Noether served as an editor of the mathematical journal *Mathematische Annalen*, helping many young mathematicians prepare their writings for publication so that the concepts they were presenting became clear to readers. She also wrote obituaries of several important mathematicians, including her *Doktorvater* Gordan (with help from her father) and later of her father himself. Although her editorial work was not mentioned in her curriculum vitae, her influence on this journal was significant. It also represented countless hours of unpaid, difficult work, for which she certainly deserved recognition. In fact, several of Noether's own writings of her original mathematical research also appeared in this important journal.

In 1920, Richard Courant was brought to Göttingen to replace Klein who was retiring as administrator of the mathematics program. Although Courant's first love in mathematics was differential equations and linear operators, his work on the mathematics program as a whole may have been his most significant accomplishment. Emmy Noether had little respect for his "administrivia," but she benefited greatly from many of his innovations. Courant worked diligently to bring formal structure to the mathematics program at Göttingen. He boldly requested permission to change the name on their stationery so that letters from the mathematics department had the heading "Mathematical Institute of the University of Göttingen." This request was approved as the first step in Courant's program, although the university administration had no understanding

The Mathematical Institute of the University of Göttingen in 2006, a vivid reminder of the grand scheme proposed by Courant (Courtesy of Dennis M. Wittmann and by permission of University of Göttingen, Mathematical Institute)

of the breadth of his grand scheme for his Mathematical Institute. Courant remarked, "They don't know how much it is going to cost them." At the same time, Noether wrote two important papers, the first on non-commutative rings (where the order in which you perform two operations changes the result) and the second on the theory of ideals in commutative rings. This was about the ascending chain condition for ideals in such rings. Ideals are certain very important subsets of a ring and the ascending chain condition says that any ascending sequence of ideals must become constant after a time. Rings with which this condition holds are now called Noetherian rings. From that point on, Noether was the undisputed leader in the field of abstract algebra. Her disarmingly simple proofs were totally abstract. She always looked at the big picture and at structural relations, and she refused to get bogged down in ordinary calculations. The horrific calculations of her dissertation were far behind her by now.

Göttingen Mathematical Institute reading room (Courtesy of University of Göttingen)

The main lecture hall at the Mathematical Institute in Göttingen (Courtesy of Dennis M. Wittmann and by permission of University of Göttingen, Mathematical Institute)

Emil Artin, who later used abstract algebra to solve both the 9th and the 17th problems from Hilbert's list of important problems to be solved in the twentieth century, came to Göttingen for the academic year 1921–1922 after completing his doctorate at the University of Leipzig. He studied with Noether for an entire year, thoroughly captivated with abstract algebra. Courant tried to recruit him as a member of the Mathematical Institute, but Artin said no, perhaps in part because of his horror at what Courant, the director of the Institute, called music. Courant played the piano enthusiastically and frequently, but with no regard for the classical tradition. Artin's music, by contrast, was pure and precise. After that year, Artin accepted a position at the University of Hamburg and pursued his studies of abstract algebra on his own. Noether enjoyed correspond-

ing with him over the years, but missed the friendly collaboration and "math talk" that they had enjoyed while Artin was still in Göttingen.

During the break between semesters, Emmy Noether sometimes went to Breslau to visit with her brother Fritz. Fritz's wife Regina welcomed her rather odd sister-in-law as graciously as she could. When Fritz and Regina were expecting their first child, Emmy was delighted. Expecting to produce no children of her own, she was pleased that Fritz and Regina's children could carry on the Noether mathematical tradition.

"Fritz," Regina asked her husband after one of Emmy's visits, "does your sister ever talk about anything but mathematics?"

"Well, I suppose mathematics is her consuming interest," Fritz admitted, "but, yes, she does have other interests."

"I find her hard to talk to," Regina said, "and she doesn't even seem to notice the food she is eating! I could have fed her canned meat and cold potatoes, and I don't think she would have minded. I worked hard to prepare nice food for her, and it seems as if I shouldn't have bothered."

"I guess food is not very important to Emmy," Fritz said. "I, on the other hand, appreciated the meals, and I'm sure she enjoyed them in her own way."

"Yes, she did appear to gobble them up!" Regina said. "Still, I find her very odd. Do you think this baby can ever live up to her expectations for him as a mathematician?

"I wouldn't worry about that," Fritz assured her. "I'm sure our baby will be happy and successful, and I know Emmy will

approve of whatever he does. Thank you for being such a good hostess to her."

"I'm just glad she doesn't visit us too often," Regina admitted.

In 1921, Emmy Noether made yet another sad trip home—to bury her father. In her usual manner, she did this without complaint and without fanfare. She and her father had always been close, joined by their shared love of mathematics as well as their father–daughter bond. Although her mathematics had been beyond his understanding for many years, Emmy's father had a general idea of what she was doing, and he could see how far it was taking her. He was extremely proud of her. She had gone where no woman before had gone, accomplishing far more in mathematics than Hypatia or Kovalevsky or Young.

Now Emmy's family consisted only of her brother Fritz, his wife Regina, and their children, in addition to her younger brother Robert, whom the family had institutionalized so many years before.

Part IV
Fräulein Professor Doctor Emmy Noether

The Noether Boys

On April 6, 1922, Noether was finally given a formal position at the University of Göttingen: *Extraordinarius* (assistant professor), a post that was officially without salary, although Courant arranged for modest compensation, which could be (but might not be) renewed annually. Even with a title and a paycheck, she still could not count on a dependable salary.

"So, *Fräulein* Professor Doctor Noether," Hilbert said to her, "now that you are an extraordinary professor, we will quite naturally expect even more extraordinary work from

David Hilbert (Courtesy of Peter Roquette)

you. Everyone knows that an *Ordinarius* (full professor) produces nothing extraordinary, and that an extraordinary professor produces nothing ordinary! I have high hopes for you, Professor!" With the title of Professor, she could legally mentor doctoral students. After 14 years of brilliant work in a revolutionary new field in which she was the recognized leader, it was certainly time! In this position at Göttingen, she was *Doktormutter* to nine PhD students, all of them working in her radical new field of abstract algebra.

An evaluation of Noether's teaching, prepared by the Mathematical Institute for the Education Ministry, provided an unbiased picture of her teaching: "Her renown as a scientist is indisputable…. While less suited as an instructor of large classes in elementary disciplines, she is capable of exercising great scientific influence upon gifted students, many of whom she has furthered considerably, and some of whom have already achieved Ordinary Professor status." There was no need to harness her to teaching elementary mathematics to novices—let others teach the introductory courses; the prestige of the Mathematical Institute would ride on the work of great minds like hers.

Noether had a close-knit group of students who often seemed like her personal guard, a bit like a family, escorting her everywhere she went, listening to her every word, lounging in her apartment in the evenings while she prepared pudding, and discussing mathematics all the time. Sometimes they argued, sometimes they pestered her with questions, and most importantly they were intensely loyal to her. These young and not-so-young men and women came to be called "the Noether Boys," while she was often referred to as "*Der Noether*," a mas-

culine form that showed the total respect everyone had for her as a first-class scholar regardless of her gender. In later years, her Russian students often followed her around in their shirtsleeves (without jackets). "Is that any way for young scholars to dress in public?" people asked. This attire came to be called the "Noether-guard uniform." Noether and her rowdy "boys" formed a major part of the colorful university life in Göttingen. One of her students, Jakob Levitski, described them as flocking around her "like a clutch of ducklings about a kind, motherly hen."

From time to time, other students were curious about this odd person whose followers found her so compelling. Everything about Noether was distinctive, from her unconventional appearance and charisma to her thoroughly abstract subject matter, which did not look like any mathematics they had ever studied! They could not be blamed for wondering what they were missing, and occasionally some of them chose to find out for themselves what her students found so captivating. When a group of them arrived at the lecture hall, they found Noether's regular students already occupying all the places in front, leaving only the back rows for the visitors. The visitors joined the Noether Boys in shuffling their feet loudly when it was time for her to begin (to signal their respect and their readiness to listen) in the tradition of German academia. Then *Fräulein* Professor Noether would begin, often picking up in the middle of the sentence she had been working on the week before. She gave no context for the lecture—the subject matter was obvious to her, so it must be obvious to her listeners as well. As she spoke, she tried to explain a difficult concept one way, but sometimes she would realize before she had fin-

ished the sentence that her explanation wasn't exactly right, so she interrupted herself to restate it in a different way. Her regular students were frantically taking down all they could in their notebooks, although occasionally one would call out a question or correction if she got muddled or lost her way. Often, after the first half hour of the lecture, one of the regulars would turn around, notice that the interlopers had quietly left, and announce with finality: "The enemy has been defeated; he has cleared out." The Noether Boys were an exclusive group, marked by their utter devotion to their *Doktormutter* and her scholarship. At the end of the lecture, once again, the students shuffled their feet loudly in appreciation of another brilliant but difficult lecture.

Noether taught courses such as Group Theory and Hypercomplex Numbers, often teaching what appeared to be the same course semester after semester. It was never, however, the same course. Noether was evolving her theories as she taught, and each time she taught a course she presented new material. Her listeners were her own advanced students, many of whom were foreign, as well as many professors and visiting scholars. Once when she arrived for class and found a crowd in her lecture hall, she cheerfully said: "You must have the wrong class! I never lecture to a large group."

"No, no, *Fräulein* Professor," they assured her. "We have come for your lecture!" These guests joined the Noether Boys in shuffling their feet before she began, and they joined the foot-shuffling again afterward. They stayed for the entire class, but afterward one of her regular students passed a note up to the podium: "These visitors have understood the lecture as well as the regulars."

This did not, of course, mean that the visitors had understood the lecture. The visitors did not have the background of Noether's regular students, and even the regulars could not understand her lectures without a great deal of independent work to clarify her completely abstract ideas. They had spent many hours preparing themselves for the lecture, and they would spend many more hours putting together the ideas that she had presented and framing questions to ask her when they next met. Hans Lewy said years later, "I have no doubt she had a very clear understanding of what she was saying, but she didn't have a clear idea of what [words] she was going to say [it in]." Noether herself once commented in a letter to her colleague Helmut Hasse that teaching difficult concepts to students was as much fun for her as for the students. She saw her teaching as a cooperative adventure. Only the most committed visitors returned to her class.

When students had questions, Noether often referred them to Dedekind. She expected them to read his *Lectures on Number Theory* in all four editions, as she had done at Fischer's urging. Unless they had thoroughly mastered Dedekind, they had no chance of understanding her work and lectures.

In a letter home to his mother in 1931, Saunders Mac Lane, an American doctoral student at Göttingen, described Noether's teaching: "[Professor] Noether thinks fast and talks faster. As one listens, one must also think fast—and that is always excellent training. Furthermore, thinking fast is one of the joys of mathematics." Noether was often criticized for talking so quickly as she lectured that a five-syllable word might be reduced to only two or three syllables! That made her lectures even harder to follow.

In general, before presenting a lecture, Noether thought through the material that she planned to present, but not in great detail. Sometimes the results were disastrous. One day when she was trying to prove a theorem using a novel approach from abstract algebra, she had to work and work at it. She struggled, she started over, she struggled, she explained. Finally, she threw down the chalk and stomped on it angrily, grinding it into dust under her heel. "Crud!" she shouted. "Now I have to teach it the way I don't want to teach it!" She then proceeded to teach it perfectly in the traditional way.

As she taught, she sometimes needed to write certain terms or equations on the board. Noether's focus was seldom on her listeners, however—it was on the topic. Once she had written the mathematics on the board, she felt she had done her part and hastened to erase it, often before anyone had had a chance to focus on it, let alone to think it through.

"*Fräulein* Professor!" one of her regular students called out. "Would it be possible for you to write that expression one more time? I believe it may be necessary to understand what you wrote before moving on to what is coming next, and I don't want any of us to miss it."

"How kind of you to be so considerate of your fellow students!" she answered with a twinkle in her eye. "Your guess that it is an important part of the lecture is correct. Otherwise, I certainly wouldn't have written it up for you at all. As it is, I guess I only came close to communicating it to you the first time. In my mind it was clear. Oh, well! Here it is. Isn't it lovely? Now would you allow me to move on?" By then, of course, she had already erased it again.

Noether had strong opinions on the subject of mathematics and how it should be presented to the young. She occasionally quoted Laplace, the great French mathematician:

"If the human mind were limited to the accumulation of facts, science would be nothing more than a list. It would show us nothing of the laws of nature."

To Emmy Noether, the patterns of nature and mathematics were there to be observed and analyzed, using all the faculties of a well-disciplined mind. Through her teaching, she strove to hone even further the intelligent, well-disciplined minds of her students.

Grete Hermann was Noether's first official doctoral student at Göttingen, earning her doctorate February 25, 1925. During the 1930s, Hermann actively opposed the Nazi regime, even teaching at a school in Denmark for the victims of National Socialism before escaping to England for the duration of the war. She returned to Germany after the war and pursued a successful career in physics and philosophy. Noether was delighted to serve as mentor to female scholars like her.

Heinrich Grell, her second doctoral student at Göttingen, remarked many times how indebted he was to his *Doktormutter* Noether. She introduced him to new ways of thinking and doing mathematics, and he followed her brilliant example as he carried the field even further. Noether was a pioneer in a field that was most certainly the future of algebra—from then on algebra would never be the same.

Rudolf Hölzer was Noether's third doctoral student at Göttingen, but sadly he died of tuberculosis between the time that he completed his dissertation and the time he would have received his doctorate. She wondered what great mathemat-

ics he might have produced if he had had more time. Under Noether's influence, the Noether Boys became some of the major players in the world of mathematics in the twentieth century, and Hölzer should have been one of them.

In the square in front of Göttingen's city hall, there is a bronze fountain portraying a young woman with a goose under her arm—the *Gänseliesel*. Since its installation in 1901, a tradition has grown that any doctoral candidate who has passed the oral examination for the PhD must climb the fountain and kiss the goose-girl. Of course, this practice is officially forbidden, but what would a PhD from Göttingen be without a kiss for the goose-girl? Exuberance was practically Noether's middle name and when Heinrich Grell passed his examination, she led the way to the fountain. "Now, boys and girls!" Noether

A PhD candidate gratefully kissing the *Gänseliesel* (Courtesy of Dr. Nazaret Bello Gonzalez and Axel Wittmann)

called. "This is a great occasion! Let's call out a mighty cheer for *Herr* Doctor Grell! Hip, hip, hooray! Hip, hip, hooray!" she shouted loudly and happily as he removed his shoes and socks, tucked his doctoral robe under his arms, climbed the slippery fountain, and happily kissed the goose-girl. Boisterous as always, Noether's strident voice could clearly be heard above the cheers of Grell's fellow students. Much to everyone's delight, she did the same thing with all her successful doctoral candidates. Certainly she was never a sedate or proper professor, looking scornfully at such high-spirited antics. That would not have been Noether-like at all.

One of Noether's favorite post-doctoral students was Bartel van der Waerden, who came to her with his degree, straight from a university in Holland. When he was a child, his father had sent him outside to play in the open air rather than allow-

Bartel van der Waerden (Courtesy of Peter Roquette)

ing him to stay inside with his beloved mathematics books because of his father's firm belief that a child needs open air and vigorous activity. While Bartel's father thought he had taken away the means to do mathematics, he had only made it difficult—he had not made it impossible. Young van der Waerden simply made up in his own mind the mathematics that he needed, inventing his own notations for trigonometry to make up for his lack of book access. When his father discovered what the child had done, he immediately returned the books to him. He felt that if Bartel was going to do trigonometry, he should do it in the standard way, with standard notation. His father could see that from this point forward nothing was going to keep his child from doing mathematics.

A decade later, when van der Waerden arrived in Göttingen, Noether quickly recognized his brilliance. Van der Waerden recognized hers, also. He took copious notes during her lectures and eventually published a textbook on abstract algebra, stating clearly that it was based on Noether's Göttingen lectures and on Emil Artin's Hamburg lectures. Published for the first time in 1931, this two-volume work was the definitive textbook in the field for many years.

Van der Waerden summarized Noether's overriding rule for mathematics:

All relations between numbers, functions, and operations become clear, generalizable, and truly fruitful only when they are separated from their particular objects and reduced to general concepts.

Although Emmy Noether was a remarkably creative person, taking a vast array of mathematical concepts and putting them together in a revolutionary way, it must be admitted that

she had difficulty explaining her constructs to the world at large. Only a select few (the Noether Boys and some of her closest colleagues) could understand her lectures well. Van der Waerden was able to present her concepts clearly in his lectures and books, and if he had done nothing else, he would be important in the history of mathematics for that alone.

The Mathematical Institute at Göttingen

The year 1923 was difficult in Germany. In the aftermath of "the war to end all wars," the Reichsmark, which had been a stable currency since the founding of the German state in 1871, had plummeted into chaos. Inflation was so extreme that money became almost meaningless. A sum of money might be worth a small fraction of its value within only a few hours during the height of Germany's hyperinflation. With a rate of inflation of 100 to 200 percent a day, working people began to be paid twice a day, rushing out to spend their morning wages on anything they could find at noon because the money would be worth only half as much at the end of the day. Only food and other concrete objects could retain their value.

People would tell a joke about a man who left an old wheelbarrow full of money outside a shop while he stepped inside for a few minutes. When he emerged from the shop, he found his money in a pile on the ground—his wheelbarrow had been stolen!

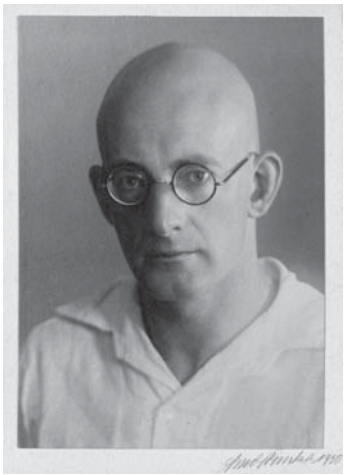
During this time, no one had any financial security, but those who knew the exchange rate a few minutes before everyone else had a slight advantage. Through a clever deal with the university, the Mathematical Institute was able to use their

futuristic calculating machine to allow the mathematicians to get the new exchange rate a few moments before the rest of Göttingen. This meant that they could buy products before businessmen raised their prices to match the current rate. This helped, but regardless it was a frustrating time at the Mathematical Institute at Göttingen. Noether, who had to live mainly off her small fixed income, was hurt at least as much as everyone else, but still her concern was only for those who had less than she had. As always, she quietly did what needed to be done, keeping the focus on others rather than on herself.

Germany's troubled economy was dramatically stabilized on November 20, 1923, with the introduction of a temporary new currency, the *Rentenmark*, and suddenly life became more bearable. Hilbert had been pessimistic about the new currency—commenting that in mathematics one cannot solve a problem by simply changing the name of the variable!—but in fact stability returned.

In 1923, P. S. Alexandroff, a prominent Russian mathematician who could both speak and write excellent German, came to deliver a series of lectures at Göttingen. Noether, who had been fascinated for years with events in Russia, was enchanted with what she perceived as the Bolshevik idealistic view of society and socialism's potential as a more humane organizing force in society. She even joined the Social Democratic party, which may have been a contributing factor in her problems a few years later, when she was labeled as a left-leaning radical.

Noether was impressed with the mathematician Alexandroff. His work in topology complemented her abstract algebra in exciting new ways, and she relished their interactions. He, in turn, recognized that she was a great mathematician



Pavel Sergeevich Alexandroff (Courtesy of the archives of the Mathematisches Forschungsinstitut Oberwolfach)

with whom he could work productively. Both Noether and Alexandroff could see how Noether's abstract algebra could contribute to Alexandroff's topology. He was impressed with her intellectual enthusiasm, with her commitment to the importance of her remarkable new ideas, and with the simplicity and warmth of her interactions with her students. To him, she was a new kind of scholar, committed to both her mathematics and her budding students. He looked forward to future collaborations with her. Noether saw mathematics as the essence of nature, not as an accumulation of discrete facts to be memorized and categorized. It was the analysis of those facts and the resultant establishment of overriding principles that was her goal. To her, mathematics was the central thread, capable of explaining everything.

The summer of 1925 was a charmed time at the Mathematical Institute at Göttingen. There were frequent algebraic-topological walks led by Noether, particularly when the brilliant Alexandroff was in residence. The assembled mathematicians spent many afternoons and evenings together, sometimes boating at the Courants' place on the Leine River or swimming at the Klie swimming pool. Although the pool was theoretically for men only, the Noether/Courant group didn't follow that rule. As they swam in the pool or rowed on the river or walked along its banks, the major focus was always mathematics.

The musical evenings were a setting for even more mathematics. With Courant at the piano and several people on a variety of other musical instruments, the tempo was quick and spirits were high. While it is true that Courant hit only about 75 percent of the notes that appeared on a page, he didn't worry about the other 25 percent, and he played with such gusto that his guests hardly noticed. Noether did not feel compelled to demonstrate her expertise on the piano—"The Happy Farmer" would not have fit in very well—and, besides, she had mathematics to discuss.

For several years Emmy Noether and Nina Courant, Professor Courant's wife, had gone for a swim every day in the Klie pool, which was more like a swimming hole than a pool. Although sometimes students had been there before them, and the water was already muddy when they arrived, they didn't worry. They just loved to swim.

"Nina!" Emmy called out. "The water is so muddy, I'll bet you won't be able to see where I am going if I dive deeply. See if you can predict where I'll resurface!"

"All right, Emmy!" Nina shouted. "I think this is where you are headed." As Emmy came up at some distance from where she pointed, Nina admitted, "Oh, my! I was off by several meters! It is remarkably muddy today! Shall we swim a few laps, Emmy?"

"Yes, indeed. I must say it's rather chilly. I need to get moving before I freeze in place!" Emmy said.

One afternoon after their swim, Emmy asked Nina to recommend a dressmaker. Nina, who had noticed how threadbare Emmy's clothes had become, was happy to give her the name of her own dressmaker, *Frau* Ebert. Emmy contacted *Frau* Ebert the next day, and the following Monday morning Emmy arrived at the dressmaker's house precisely at ten o'clock.

When *Frau* Ebert showed her drawings of some new stylish dresses, Emmy Noether was delighted. These new styles were what she had been trying to wear for years—no discernable waist, nothing constricting to her arms, just a simple dress that dropped from the shoulders to the hem. Noether ordered three new dresses, and she was pleased with the results. Although they didn't look particularly stylish when hung on her large frame, they were presentable and they were comfortable.

"Well, Nina," Emmy asked her friend several weeks later, "what do you think of my new frock?"

"I think it's lovely!" Nina said. "You know, it is the latest style!"

"That's what *Frau* Ebert said," Emmy blushed.

"Haven't you seen what everyone is wearing now?" Nina asked. "*Frau* Hilbert was wearing a dress very similar to this one last week when I saw her."

Emmy hadn't noticed a thing.

Van der Waerden's visits in Göttingen always occasioned some festive mathematical discussions, both in Noether's "mansard dwelling" and on free-ranging walks through the town, with mathematical scholars of every imaginable level from Hilbert and Courant down to the lowliest student, all participating on an equal footing. Those relaxed discussions, often warmed by pleasant drinks and snacks, were part of the atmosphere that permeated the Mathematical Institute of the University of Göttingen" in the late 1920s and early 1930s. The Noether school was clearly the most fertile and exciting scientific salon in Europe at the time. Students and scholars alike perched on odd pieces of furniture or on the floor in her apartment to explore mathematics at many different levels. They occasionally revisited old concepts in radical new ways and other times broke out in new directions. Contributions from the young were accepted just as readily as the suggestions of the professors. Emmy Noether could be counted on to share ideas from her current research, and many a student went home from these evenings with an idea for his all-important thesis. This was mathematics at its best.

During the winter of 1926, Noether and Alexandroff met again in Holland, where he was working and lecturing. Alexandroff was focusing on continuous partitions of topological spaces, and Noether was continuing to develop her notions of set theory in abstract algebra. They both could see that the topics complemented each other perfectly, and they both enjoyed the collaboration. Soon, Alexandroff was so enthused with Noether's abstract algebra that he was delivering lectures

in that field in Russia. Eventually, he recruited A. G. Kurosh as a doctoral student for Noether in Göttingen.

In 1928, Emmy Noether delivered an address at the International Congress of Mathematicians in Bologna. Once again, she was recognized as one of the foremost mathematicians of her day. She still had no "ordinary" professorship complete with generous compensation at Göttingen, but the mathematical world stood in awe of her accomplishments. She was never honored with membership in the Scientific Society of Göttingen but she was not alone in that. Many esteemed professors in Göttingen have failed in that.

Also in 1928, Emmy's youngest brother Robert died in the state institution for the handicapped where he had spent his last few years. His family had been careful to provide the care he needed, but his had been a sad life. He never understood anything of the gifted family he was born into, and he knew nothing of the amazing accomplishments of his older sister, although she had long used her limited money to support him in the institution until his death. She shed a tear for him on his grave, but who could tell whether it was in sadness or in relief?

Emmy Noether: A Respected Scholar

In 1928, Soviet Russia was beginning its notorious five-year plan under the newly triumphant dictator Josef Stalin. The program was advertised in glowing terms, and, like many liberal intellectuals outside Russia, Noether was ready to accept its claims at face value. When she traveled in Russia, her myopia allowed her to miss the many signs of a repressive regime committed to industrial and agricultural progress at all costs. She never knew about the dissidents who were exiled to Siberia or the peasants who were literally starving on their own farms during the forced collectivization of agriculture. Although the crops the peasants harvested would fill someone else's belly, they were powerless. Seeing what she wanted to see, Noether respected the Soviet state's superficial commitment to idealistic socialism—from each according to his ability, to each according to his need. Her friends in Göttingen shook their heads in disbelief at her naiveté. How could she possibly be so deceived by Soviet propaganda? Despite her amazing intelligence, she was blind to the contradictions of the Soviet system.

During the winter of 1928–1929, Noether was a visiting lecturer at Moscow University. While there, she lived in the student dormitory, walking each day over the Krymskii Bridge

to the university. She was eager to learn about student life in Russia's "ideal" society, and spent hours conversing with any students she could find who were able to communicate with her in German.

"Excuse me, sir," Noether began one day as she crossed the bridge next to a young student, "do you speak German?"

"Yes, I can speak some German," he replied tentatively.

"Oh, good," Noether said. "I am from Göttingen in Germany, and I am spending a semester here at the university. I am fascinated by student life in Russia. Can you tell me where you studied before you came to Moscow?"

"I studied in my village until I achieved my school-leaving certificate," he explained. "My teacher thought I had some talent, so he urged me to submit some papers to the university here, and they invited me to come and study. That was three years ago. At the end of this year, I hope to complete my PhD.

"You're an ambitious young man, in all the best senses of the expression!" she congratulated him. "Have many people from your village been able to do as well as you?"

"Yes," he said. "A couple of them have been able to complete the doctorate, and they have been able to do great things.

"So only two or three have been able to do that?" she asked.

"There may have been more," he answered. "I really don't know. I do know that I am fortunate to have this opportunity."

In 1929, the Mathematical Institute at Göttingen moved into a new building funded by the Rockefeller Foundation in New York. It was a wonderful facility, built to provide the perfect setting for mathematical inquiry. In the basement were a room for bicycles, a book bindery, and a small canteen for snacks. On the main floor were two auditoriums, one large and the other far larger, for mathematical lectures. At the front of the lecture halls, the many blackboards were on a sophisticated pulley system that enabled a speaker to write some formulas, move on to the next board, and then the next, while all three boards remained visible from the auditorium seats. On the second floor was the most important part of the facility: the *Lesezimmer*—reading room—with all the print resources that a mathematician might need gathered conveniently in one place. This was Klein's innovation, something that Göttingen mathematicians have benefited from ever since. The building also included offices, a radical new feature for a university department, and these were assigned to individual faculty members. Up to this time, the only office a professor had was in his own home. Noether's office at the institute, one of the largest, was often loud with the animated discussions of the Noether Boys and their mentor.

In 1930, Noether met Olga Taussky, who had grown up in what is now the Czech Republic and was just finishing her degree at the University of Vienna. They were both attending a meeting of the Deutsche Mathematiker-Vereinigung at Königsberg, and Noether listened eagerly to Taussky's presentation. Afterward, both Noether and her younger colleague Helmut Hasse commented at length on Taussky's paper. Taussky was delighted that these two important mathemati-

Olga Taussky-Todd (Courtesy of the archives of the Mathematisches Forschungsinstitut Oberwolfach)

cians were genuinely interested in her work, although at that time some of their comments were beyond her understanding. Noether's friendship with Taussky was important to Noether for several reasons, but at first Taussky's main appeal was that she was a female mathematician, a breed that Noether was predisposed to admire. Noether was very pleased that other women were finally beginning to be accepted as mathematicians. At this point, she had seen only one woman through to the completion of her PhD—Grete Hermann; the majority of the Noether Boys were, in fact, young men.

At the conference luncheon, Taussky sat at Noether's left while several other mathematicians sat at her right and across the table from her. Noether had a magnificent time, eating and talking with irrepressible enthusiasm. Observing her, Taussky couldn't help noticing Noether's table manners. While chew-

ing, Noether often talked with both hands, gesticulating wildly to make her point clear. Since eating was not Noether's major focus, gobs of gravy or bits of potato appeared on her ample bosom from time to time. When she noticed, Noether casually brushed them away, never stopping her conversation to make sure that the spot was gone. At the end of the meal, the menu was well displayed on the bodice of her dress, but as usual Noether was not concerned. She had enjoyed a delicious lunch and animated conversation.

David Hilbert was now 68 years old, the age at which a professor must retire according to German law. Hermann Weyl was invited to return to Göttingen as an *Ordinarius* to fill Hilbert's place at the university where Weyl had studied so many years before. As he claimed his esteemed position, he became acutely aware that Emmy Noether, whom he considered a far greater mathematician than himself, was still the lowliest of assistant professors.

Hermann Weyl (Courtesy of Peter Roquette)

"Gentlemen!" he complained. "How can this be? She is the greatest living mathematician today. She must be made an *Ordinarius* immediately!" His efforts on her behalf were futile, however. She had too many strikes against her: She was a woman, she was unconventional, she was a Social Democrat with strong communist sympathies, she was abrasive, and she was loud. Professors in Germany were nothing like that; Weyl reluctantly conceded defeat for the time being.

In 1931, Emmy was teaching happily in Göttingen, making many extraordinary discoveries despite her lowly extraordinary professorship. The Noether Boys were doing well, and Emmy was pleased with their progress. She also knew that she was successful as a scholar, and she could see that her ideas were being accepted widely in the mathematical world. She was truly in her prime. As she lectured, when a particular point came across just perfectly, she would sometimes stop a minute and look shyly at her audience, smiling modestly like a little girl who had just accomplished something particularly nice. Her audience was enchanted.

Emmy's style of lecturing was unusual in many ways. She had a particular manner of dealing with her handkerchief during class that some observers found amusing. Although everyone occasionally needs to use a handkerchief, women's clothing in the 1930s was not constructed to provide easy access to this item. There were no pockets in women's clothing, so Emmy kept a handkerchief tucked inside her blouse. When she needed it, she would pull it out with a flourish, use it quickly, and then tuck it back into this discreet spot. Her students noticed that it didn't take long for her blouse to become somewhat askew, but she had already moved on to her next thought

and never noticed. Some observers tracked the handkerchief's progress as she used it repeatedly during a lecture, occasionally losing it entirely in the deep recesses of her blouse!

Having allowed her hair to grow long once more, finding it more convenient than short hair, she pinned it up so it would not interfere with her lecturing. As she lectured, however, her hair sometimes came loose from the hairpins, yielding a rather wispy look, which became looser and looser as she talked. Since she couldn't see it, and it wasn't in her way, she was happily oblivious to her now-disheveled appearance. When a young woman in her class approached her during the break one day to offer to help repair her hair, Emmy was so caught up in a discussion of one of the points in the lecture with another group of students that the helpful young woman never even got a chance to speak.

German life has always had room for holidays, which Germans as a whole celebrate enthusiastically, but such holidays were sometimes ill-timed for Noether. On more than one occasion, students reminded her that the next class meeting would have to be cancelled because the Institute would be closed for a national holiday. Unfazed by such inconveniences, she would announce, "Well, all right then, we will meet at the usual time but on the steps in front of the Institute. We can easily proceed with class from there." When they met that day, she might lead them on a walk to the outskirts of town, where they would find a pleasant spot in the shade of some large chestnut trees. Or the professor and her "Boys" might follow a path through the woods, climbing over fences as needed, and arriving at a coffee house to enjoy a cup of strong coffee and a piece of cake before returning home. Either way, a lively mathematical

discussion proceeded from beginning to end without interruption. None of the Noether Boys ever considered complaining about losing their holiday. This was classic Noether, and they were addicted. How else could they master the difficult and abstract field that she alone could present to them?

With her own students, Noether was an especially warm and loving mother. Part of her genius was the fact that she would spend as much time as a student might need to grasp a difficult concept. She nurtured her students lovingly, and they adored her in return. Of Noether, van der Waerden wrote succinctly, "She was both a loyal friend and a severe critic."

Once when Noether was out walking with her students, discussing mathematics with them happily, it began to rain—not a surprising event during a German winter. When she pulled her umbrella out of her satchel, her students couldn't help but notice that it was in tatters. It was hardly worth opening! One student commented that she really needed to have it repaired, but her cheerful reply closed the conversation effectively: "I need it only when it rains, and when it doesn't rain I never think of it."

On December 6, 1932, known as St. Nicholas Day, when St. Nicholas brings treats to good little boys and girls, the door suddenly burst open in the middle of her lecture, and someone brought in a large cake inscribed with a message about the Noether Boys.

"Who do you suppose provided this cake?" Noether asked.

"It must have been Hasse!" someone replied. This would not have been surprising since Professor Helmut Hasse, a younger scholar with whom Noether was working closely, was

very enthusiastic about her work and enjoyed contact with her vibrant group of disciples. The cake was quickly cut and consumed, and the students agreed to send an amusing thank you note to Hasse. Noether even wrote a silly addition to the note herself. Hasse was stunned. In fact, it was the Noether Boys themselves who had provided the cake. They enjoyed their joke (as well as the cake) hugely.

Recognition as a Scholar

In 1932, Noether taught in Frankfurt for a semester. There, too, students and faculty respected her for the major scholar that she was, and she enjoyed the approval she received. At that time, she and Emil Artin together won the Ackermann-Teubner Memorial Prize of 500 *Reichsmark* (*Reichsmark* had been reinstated as the name for German money in 1924) in mathematics for the sum of their scientific accomplishments. She knew that her mathematics was important, she was pleased to share the prize with her colleague Artin, and it was gratifying to have their work acknowledged in the mathematical community. Although money was never very important to her, it was nice to receive this acknowledgment. It seemed that her career was moving along beautifully.

Once again, Noether spoke at the International Congress of Mathematicians in September 1932 in Zurich. Approximately 800 people attended the congress, and Noether was one of the 21 keynote speakers and the first woman ever to give a major talk at that meeting. Clearly she had arrived: she was recognized as the leader in her field. One of the high points of the congress was a dinner hosted by a Japanese mathematician, Teiji Takagi, at the Hotel Eden on Zurich's beautiful alpine

lake. Everyone who was anyone in class field theory attended the dinner, including Emmy Noether and Olga Taussky. The Japanese participants, who could hardly believe the rowdiness of the crowd, were amused by the social interactions of European mathematicians. During the meal, as Noether gesticulated wildly to emphasize some point she was making or to signal to someone else that it was her turn (once again) to talk, the Japanese saw an unusual example of European table manners as well.

As the evening reached an end and the guests were preparing to return to their hotels for the night, Noether begged her hosts to teach her to bow in the Japanese style. As she bent forward at the waist, she asked for pointers: "Should I do it like this?" she asked, behaving in much the same way as she had at evening dances when she was still a schoolgirl in Erlangen. Since she was the senior mathematician at the party, no one laughed outright at her performance, with the exception of Noether herself—whose guffaws rang out loud and clear!

Before the congress, Noether had gone on a short vacation in the Alps in the town of Wengen, near the dramatic Eiger Mountain. The trip from Zurich required that she take a series of trains. First, she took a standard train to Luzern, then she transferred to a cog railway train as far as Interlaken, then to a smaller cog railway up into the Berner Oberland, and finally to an almost-miniature cog railway, taking her high up to the Alpine village of Wengen. In this idyllic setting with its pristine mountain air, she could walk and engage in "math talk" to her heart's content. She and some mathematical friends, who stayed in a quaint hotel in town, loved wandering around the village looking at the cozy chalets that made up the town. She

A cat ladder in an Alpine village (Photograph by the author)

chuckled when she found a long, narrow ramp leading from an upstairs windowsill down to the sidewalk below, clearly constructed for the convenience of a cat who lived in the upstairs apartment. This cat would not be denied the occasional breath of fresh mountain air whenever he wanted it!

One day, Noether and a few companions took a picnic lunch with them as they wandered up toward Kleine Scheiddeg, one of the shoulders of the mighty Eiger Mountain. Around each bend in the path were spectacular views, and Noether found her ideas soaring as high as the massive Alps that surrounded her. This was utopia as long as she could stop to catch her breath after every few steps. Noether was enchanted by the wild alpine flowers in bloom, sometimes right next to a patch

of snow left after a recent storm. Winter comes early at that altitude.

One of the big events of 1932 was David Hilbert's 70th birthday. Hilbert, who in 1900 had outlined the challenges for the mathematical community during the twentieth century, was generally recognized as the world's most important mathematician. His birthday was the occasion for serious celebrations, including the formal presentation of the first volume of Hilbert's collected works from Springer, his publisher, which Olga Taussky was editing.

In contrast, Noether's 50th birthday a few months earlier had not been viewed as a major event although her friend and colleague Hasse celebrated it by dedicating a paper on deriving a reciprocity law using only non-commutative algebras to her. The subject was close to Noether's own research, and she was delighted. Wolfgang Krull, one of the Noether Boys who was now a professor of mathematics at Noether's alma mater in Erlangen, was heard to comment at the time, "*Fräulein* Noether is not only a great mathematician, she is also a great German woman!"

At this time, Noether was working with Robert Fricke and Oystein Ore to edit the collected work of Richard Dedekind, Noether's mathematical hero. She had used Dedekind's work as a point of departure for her work in abstract algebra, often telling her students, "you will find it in Dedekind," no matter what "it" was. Noether did much editing to the work herself, adding explanatory comments to clarify Dedekind's work. With help from Jean Cavaillès, she also edited the complete

correspondence of both Georg Cantor and Dedekind. In the foreword, Cavaillès praised Noether for her "intense intellectual radiance."

Many gifted mathematicians from all over the world came to Göttingen to study with Noether, including many Chinese and Japanese mathematicians. Chiungtze Tsen completed his PhD in Göttingen in December 1933. Teiji Takagi, Hilbert's student who had entertained Noether and her colleagues so generously in Zurich, sent his student Kenjiro Shoda to study with Noether in Göttingen. Shoda's textbook on abstract algebra brought modern algebra to the universities of Japan in the next few years. For the rest of his life, Shoda always kept a picture of Noether on his office desk in Tokyo.

The French also were caught up in the enthusiasm for abstract algebra that was rapidly developing in Göttingen. Because very little new mathematics was being done in France at the time, eager French students flocked to Göttingen. The most impressive French mathematics at the time was the work of the "mathematician" Nicolas Bourbaki. In reality, "Bourbaki" was a group of young French mathematicians in the 1930s who set out in a light-hearted way to bring mathematics in France back to life. The Bourbaki work on abstract algebra was actually based on van der Waerden's text, which in turn was based on the ideas of Emmy Noether and Emil Artin.

In the mathematical culture of 1920s Göttingen, there was one underlying thread of controversy that seems to have arisen from David Hilbert's view that saw the current state of mathematics as a body of work belonging in common to all mathematicians. By the time they published, the older mathematics had been transformed at least in part into new and

original ideas, and they saw no need to defer to whoever had had the first inspiration. After all, no mathematician anywhere felt compelled to give credit to Euclid when he or she made a discovery in geometry; should a mathematician have to list hundreds of sources when publishing? Of course not! It was a process facetiously called "nostrification"—the process of taking what is yours or theirs and transforming it into what is now ours, based on the Latin word *noster*, meaning "our." This tendency was recognized in the context of the foundational crisis, involving the Dutch mathematician Brouwer, and also by Einstein who used the term in a letter disputing Hilbert's claims on relativity theory.

In Noether's school, there was something else at work. Noether's mind was creativity embodied. She was always coming up with and exploring new ideas, taking them in directions that no one else had ever considered. Since she did much of this work in the company of her students, they were present at the birth of many significant new concepts. Instead of guarding her ideas jealously, however, she eagerly passed them on to her students, urging them to take this or that idea and develop it into a paper or dissertation. Following Göttingen tradition, one could call this "vestrification"—the process of making what is mine into what is yours based on the Latin word *vester*, meaning "your." As someone almost devoid of ego, Noether was excited to hand off her new ideas to one eager student after another and to watch their development. She might well be involved in the refinement of the ideas, but still the "vestrification" was intact: the work was still the property of that student.

Noether enjoyed her "mansard" apartment for many years, maintaining a good relationship with her landlord, a Professor Kees, whose specialty was Egyptian history. As politics in Germany were changing and with the deepening of the Great Depression, however, Noether's comfortable setting could not last. Student members of the *Turnerschaft Albertia*, an ultra-conservative student fraternity that also rented rooms there, protested Noether's presence in "their" building in 1932: "We are not willing to live under the same roof as a Jewess, particularly one who has such sympathies for the communist lifestyle in the Soviet Union! Communist, Pacifist, Jewess, depart!"

This was the beginning of the end of Noether's happy life in Göttingen. National Socialism (the formal name of Nazism) loomed ever more powerfully in German society. Although she quickly found lodging in another building several blocks away, it was clear that life was becoming difficult for her as a Jew in Germany. Even some of her friends and colleagues seemed to be avoiding her. When Ernst Witt, one of the Noether Boys, came to class wearing Nazi clothing, she said nothing, continuing to teach him and all other students who chose to study with her. Her field was mathematics, and it had nothing to do with Nazis. There was no doubt, however, that the stable German state, with its well-established and fair laws under which she had thrived for 50 years, was suddenly crumbling around her. Her persecution as a female scholar 25 years earlier was starting to look remarkably benign.

Part V
Exile

A Move to Bryn Mawr

On January 30, 1933, when Adolf Hitler became chancellor of Germany, more students and professors at the Mathematical Institute were seen wearing swastikas and brown shirts. On March 13, Hitler pushed his Enabling Act through the *Reichstag*—the parliament—giving him power independent of the *Reichstag* and demanding a nationwide boycott of Jewish businesses a few weeks later. On April 7, the *Reichsgesetze*—imperial law—decreed that non-Aryans (German Jews or other unwelcome minorities) would no longer be allowed to work as civil servants. Since university professors were classified as civil servants, they too were targeted. Although the imperial law specifically excepted anyone who had fought for Germany or its allies in World War I, even that exception was not honored.

The utopia that was the world of mathematics at Göttingen suddenly ended on April 13, 1933, when Courant, Emmy Noether, and four other Jewish professors in the Mathematical Institute received telegrams placing them on academic leave effective immediately. Courant, who had served bravely in the war, should have been exempted, even by the standards of the new Nazi law, but he wasn't. As the director of the institute,

he would have to be replaced, and that was the final blow. No one could believe it. These professors were Germans and remarkably productive members of society. Although they were some of Germany's most important scholars, they were suddenly told that they were not Germans at all. No fewer than 18 mathematicians, most of whom were Jewish, left the Mathematical Institute at Göttingen that year.

When news of the dismissals appeared in the local newspaper, Noether still couldn't believe such a thing was possible, and she refused to take it seriously. Her professional life had always been a struggle, but she had always kept on with her work and somehow things had worked out. She could see no reason to think that it would be any different this time.

The professors wrote letters in their own defense. Their colleagues wrote letters in their defense. A hundred years earlier, during Gauss's time, the incident of the Göttinger Seven had forced the exile of seven important professors, but modern Germany could not be so capricious with its scholars. In response, the Nazi government pointed out that the Mathematical Institute in recent years had been "jewified," using that spurious claim as a justification for its actions in 1933. While it was true that there were many Jews at the institute, the only credentials that mattered at that center of scholarship were mathematical. The entire nation was caught up in an anti-Semitic frenzy, however, which was difficult to fight. To the amazement of many at the institute, Helmut Hasse, Noether's friend and colleague, attempted, albeit unsuccessfully, to join the Nazi party. He was a loyal German (although, as the Nazis saw it, he had at least one questionable ancestor), and he may

have believed that joining the party could improve his position with the authorities. Nevertheless, he was a first-rate mathematician, and, after her departure from Göttingen, Noether continued to correspond with him.

Ever the optimist, Noether soon learned that she was wrong about the crisis. This time was different, and she found herself forced into exile only a few months later. Her conversion to Christianity in 1920 counted for nothing now. National Socialism's attacks on Jews were not based on religious belief—they were based on so-called racial identity. This was an ethnic distinction, and since Noether's family was ethnically Jewish, the Nazis considered her Jewish too.

Many of the affected professors received invitations to move to various American institutions, and most were grateful to go. In contrast, Noether hoped to find a position in Moscow with help from her friend and colleague P. S. Alexandroff, but the logistics moved too slowly. Somerville College at Oxford made an unsuccessful attempt to arrange a position for her with outside funding from the Rockefeller Foundation, but only Bryn Mawr, a women's college in Pennsylvania, was able to offer her an actual position (also with help from the Rockefeller Foundation). The stipend for the one-year term was subsequently renewed for two additional years. Noether, who had cabled her acceptance to Bryn Mawr on October 2, 1933, went by train from Göttingen and sadly boarded the Bremen, a beautiful modern German passenger ship bound for New York. A week later, she was walking on a lovely American college campus, surrounded by beautiful fall foliage on the outskirts of Philadelphia, Pennsylvania. She had never even heard of Bryn Mawr College until July of that year.

The Noether Boys who had not yet completed their degrees needed new advisors. Ernst Witt, who had been so bold as to wear Nazi clothing to Noether's class, had begun work on his doctorate at Göttingen under Noether in 1931. Noether noted at the time that his work was a serious attempt to take abstract algebra further and deeper, not just to clarify and refine what had already been done, and he had essentially finished it before her dismissal at the hands of the Nazis. She immediately handed Witt over to Anton Herglotz, a non-Jewish mathematician. Witt completed his PhD under Herglotz in July 1933, when Noether was still packing to leave. Otto Schilling was Noether's last doctoral candidate at Göttingen. After finishing his PhD at Marburg, he also emigrated to the United States.

One evening that winter, David Hilbert found himself seated at a banquet next to the new Nazi Minister of Education, Wilhelm Rust. When Rust asked Hilbert how mathematics at the University of Göttingen was progressing now that the Jews had been cleansed from the faculty, Hilbert replied, "Mathematics in Göttingen? There is really none anymore." That is not an entirely fair assessment. Hasse and Landau and a few others were still there, but the Mathematical Institute was basically dead. Rust apparently chose to ignore Hilbert's acid remark.

Noether's move to Bryn Mawr College, an excellent but small women's college in Pennsylvania, was the best move she could have made at the time, although a primarily undergraduate college would certainly not have been her first choice. As the world's expert in abstract algebra, she had no interest in

teaching introductory calculus to classes of freshmen, regardless of how bright and eager they might be.

The mathematical world was eagerly awaiting Noether's next topic of research. Although she had left her Noether Boys behind in Göttingen, and she didn't expect to find their peers at Bryn Mawr, the situation there was not as bad as she feared. Bryn Mawr has always had a graduate program in mathematics. Anna Pell Wheeler, the chair of the mathematics department, was a serious mathematician whom Noether respected. In fact, Wheeler had studied at Göttingen under Hilbert and Minkowski in 1907 before returning to the United States to complete her PhD at the University of Chicago in 1918. In time, Noether developed good relations with a new group of "Noether Boys," who this time turned out to be very bright young women. Like her students in Göttingen, they rapidly became as devoted to their "mother" as the "boys" in Göttingen had been.

In December of 1933, after seeing that Noether was able to communicate in English (her schooling in English back when she was 16 years old was finally paying off!), and eager to present her to the American academic world, the president of Bryn Mawr invited a dozen mathematicians from area colleges (including the University of Pennsylvania, Princeton, and Swarthmore) for a lecture by the famous Professor Noether. Recognizing that Noether was a lecturer who would be able to speak best to a carefully chosen group, the president skillfully assembled a distinguished group of scholars who she hoped would have the background necessary to understand Noether's complicated subject. The assembled scholars were impressed. Although the lecture was planned to initiate a more

formal teaching program for Noether in the new semester, it also clearly demonstrated to the assembled mathematicians that Bryn Mawr was at the forefront of modern mathematics. "Catching" Professor Emmy Noether was a real coup for this small but respected college.

The Rockefeller Foundation had also provided funding for a formal alliance between Noether and the Institute for Advanced Study at Princeton, where she delivered a weekly address. The audience there was on a par with her audiences in Göttingen, and she enjoyed her interactions with them. Olga Taussky, with whom Noether had worked in Göttingen, had been given a post-doctoral appointment at Bryn Mawr following Noether's recommendation, and she often took the train to Princeton with Noether for the weekly lecture. The two women enjoyed speaking German together and consulted earnestly about the lecture that Noether had just presented at Bryn Mawr and how it should be adapted for Princeton.

"Emmy," Olga said one afternoon, as they found seats on the train to Princeton, "you will never believe what I have found in New York: a chocolate shop that has genuine German chocolate," Olga said.

"In New York?" Emmy asked.

"Yes. You can smell it almost a block away! I bought one small piece, and it was wonderful. Would you like me to get some for you?"

"Oh, yes!" Emmy said enthusiastically. "I would love to take some real chocolate to the Brauers in Princeton. Remember, they are here in America for the same reasons that I am,

and they must be homesick too. Hershey's chocolate simply doesn't have the flavor of good German chocolate. Do you think the problem is that they put too much sugar in it?"

"That may be part of it, but it's not just that," Olga said. "German chocolate is so rich and smooth."

Another passenger sitting near them interrupted this conversation, "Excuse me, ladies, but why are you speaking German? Don't we have enough German influence in the world today without bringing it onto the local trains?"

"Pardon us," Emmy responded in English, "we are German refugees. We have been exiled from our homeland unreasonably, but that does not mean we shouldn't still speak German. It is our native language, after all, and all people express their thoughts best in their native language. You can be sure that both of us hate Hitler at least as much as you do."

"Then I think you should speak English." At that their interrogator arose, picked up his things, and moved to another seat further back in the car. Reverting to German, Emmy asked Olga, "How can people be so rude? It was none of his business what language we are speaking. If he wants to listen in on other people's conversations, I say he is simply rude."

"Maybe he was just upset that he couldn't understand what we were saying," Olga suggested.

"That could be," Emmy admitted, "but this is a free country and we should be allowed to speak whatever language we choose."

Several weeks later when they were once again in the train on the way to Princeton, Olga presented her prize chocolate to Emmy, who took one look at it and gasped. "Olga! This is

children's chocolate! Why did you buy children's chocolate for the Brauers?"

"That's not children's chocolate," Olga protested. "It's fine German milk chocolate."

"I was expecting dark chocolate, Olga. I can't believe you did this to me," Emmy said. They rode the rest of the way in silence. Olga wondered how Emmy could be so upset about chocolate and never even acknowledge her gift. The chocolate had been expensive, and Olga was not in America on a generous professorship. Her post-doctoral grant was modest.

In Princeton that evening, when the women arrived at the Brauers' for supper, Emmy apologized profusely as she presented the chocolate. "I'm so sorry about the chocolate!" she explained. "I had intended to give you excellent dark chocolate. Please forgive me."

"But no, Professor Noether!" Mrs. Brauer explained. "Maybe I never grew up, but milk chocolate is still my favorite! I know there are people who prefer dark chocolate, but I'm afraid that is not my choice. Sitting here comfortably in America we may not approve of the things the German government is doing now, but I must say I do miss good German chocolate. Thank you very much!"

Another afternoon on the way to Princeton, Emmy surprised Olga by criticizing the dress she had worn to Hilbert's 70th birthday party back in Göttingen. "Olga, I was thinking about that dress you bought to wear to Hilbert's party," Emmy chided. "I can't believe you were so extravagant! Were you trying to impress people?"

"No, I was not!" Olga exploded. "The dress was not expensive. I found it on a reduced rack for a very reasonable

price, and it seemed like a dress that I could use many times. I have to wear something, and when I find something that I like for a good price, I buy it."

"It looked too rich," Emmy persisted. Because clothes meant little to Emmy, she was appalled at the thought that Olga, whom she both liked and respected, could waste money on clothes.

"Well, it wasn't extravagant," Olga said—and she was not happy.

Emmy had another complaint too. "Olga, why are you wearing that ridiculous Austrian hat today?" she asked.

"I like it," Olga protested. "I spent several years studying in Austria, and this hat brings back pleasant memories for me."

"Do you want to advertise to the world that you are German?" Emmy asked with a sneer.

"Emmy, it's Austrian, not German," Olga explained.

"At this time, with America almost at war with the German-speaking world, I would be ashamed to wear such a hat!" Emmy announced. "I'm ashamed to be seen with you wearing that hat."

Olga was amazed at Emmy's reaction, but she never wore the hat again.

Although she was understandably upset by these incidents, Olga tried to understand. She realized that Emmy wanted desperately to do everything correctly and that her entire world had been savagely overturned. She tried to forgive Emmy, although Olga too had been disturbed by all the political changes in the world. Emmy had never been this unpleasant earlier. Olga wondered if it was possible that Emmy wasn't feeling well.

After Noether had taught at Bryn Mawr for one year, she returned to Göttingen for the summer of 1934, thinking she could continue her work as before and still unwilling to believe that her country had completely disowned her. First she went to Hamburg, where she had a wonderful visit with her former post-doctoral student and now-colleague Emil Artin and his wife. What fun it was for her to work with him again! Artin had extended several important parts of abstract algebra in the years since he had left Göttingen, and he and Noether always enjoyed discussing their new discoveries. Their styles of working were totally different, however.

"Noether, what did you just say?" Artin asked after Noether had described a new discovery in rapid-fire German as they walked around Hamburg. "I really didn't understand any of it. Please explain it again."

Noether explained her new idea once again, and once again Artin interrupted her. "Noether, I still didn't understand you. Could you please try to explain it once more?" All through this discussion, they had been walking at a brisk pace. Only when she got tired did Noether finally begin to slow down, talking more slowly and deliberately, and finally Artin understood.

"Oh!" Artin said in surprise. "So that's what you were saying. Thank you. That makes perfect sense now. What if we applied this to another ring? It will still apply, won't it?"

Most of her experiences that summer, however, convinced Noether that she could not remain in Nazi Germany. Many of her former colleagues shunned her, fearing trouble if they

were seen talking, let alone collaborating, with her. She sadly decided to remove her possessions from Göttingen. Up to this point, she had kept her apartment in Göttingen, in the hope that she could return some day. Now she realized that that would not be possible. Noether made arrangements to ship her furniture and books to Bryn Mawr, where she would be able to enjoy her own little piece of Germany.

Next, she set out to see her brother Fritz, who was still living in Breslau. He had been forced to retire before the age of 50 because the Nazi government no longer allowed him, as a Jew, to teach. Fritz's four years in the trenches fighting for Germany in the war apparently counted for nothing.

Emmy found her brother's family in despair. Her sister-in-law Regina was not coping at all well, the children were being harassed in school because they were half-Jews, and it was clear that the family needed to emigrate as quickly as possible.

"Gottfried," Emmy asked her younger nephew, "tell me how things are going in school."

"Not very well," Gottfried admitted. "I'm not allowed to speak in class, and some of the other children taunt me for what they call my "Jewish nose" in front of the whole class. I don't have any friends anymore."

"Doesn't the teacher correct those rude children?" Emmy asked in horror.

"No, I think she agrees with them," Gottfried said. "I really don't like school."

"Not even mathematics?" Emmy asked.

"Yes, I still like mathematics," Gottfried said, "but since I am not allowed to ask questions, even that is sometimes difficult."

Emmy realized that there was nothing anyone could do. Her beloved Germany had become barbaric.

Now all they could do was wait for arrangements to be made for the move to Tomsk in Siberia. Sadly, it was the last time Emmy was to see her brother and his family. The summer that she had been looking forward to had turned into a nightmare.

When Fritz finally emigrated to Russia, the original plan had been for his sons Gottfried and Hermann to stay in Breslau with their non-Jewish mother and finish up their schooling there. By then, however, they all realized that as half-Jews the boys would not be allowed to do that, so the boys went with their father to Tomsk, learned the Russian language, and completed their preparatory schooling there. Their mother, apparently finding the stress overwhelming, remained in Germany and died an unhappy woman. In 1937, since life for Germans living in Stalinist Russia had also grown increasingly difficult, Fritz's two sons left Russia and went to Sweden. Hermann, Fritz's older son, died in 1942 in the middle of the war, but Gottfried was eventually able to immigrate to America and has had a distinguished career as a mathematician and statistician at the University of Connecticut at Storrs.

Emmy Noether's brother Fritz attended a meeting of the Moscow Mathematical Society in Moscow in 1935 when Alexandroff recognized Fritz as he honored his sister Emmy. A year later, in 1936, Fritz made a 14-day journey from Tomsk to a meeting of the International Congress of Mathematicians in Oslo, where he gave a lecture that lasted no more than

20 minutes. That evening, the Congress participants were treated to a smorgasbord of food and drinks, starkly contrasting with the little available in Stalinist Russia. After the Congress, Fritz returned to Siberia, never to be seen outside of Russia again. He was put in a detention center before World War II began, accused of being a German spy trying to discover whether German U-boats could navigate up the River Ob, the largest river in Asiatic Russia. Since German U-boats never tried to navigate the mouths of any rivers anywhere because, in addition to the limited depth, the mixture of salt and fresh water there made for unpredictable buoyancy, the charges were totally irrational. Fritz was convicted of those trumped-up charges and apparently died in 1941, the victim of two dictatorships (Hitler's and Stalin's).

In 1988, more than forty years later, the Soviet Ambassador to Washington made the formal announcement that Fritz Noether had been falsely accused and falsely convicted, and his name was cleared. Emmy was probably fortunate that her invitation to teach in Moscow had been delayed. Improbable though the accusation would have been, she too might have been seen as a spy for the Germany from which she too had been forced to flee.

At one of Noether's weekly lectures at the Institute for Advanced Study in Princeton, an acquaintance from Göttingen (probably another Jewish refugee—there were many in the United States at the time) congratulated her on finally earning enough money to live a little more comfortably. He knew that her lowly status at the Mathematical Institute at Göttingen had

been a scandal, and he saw that for the first time in her life she had a regular academic position with a regular salary. Noether quickly corrected him: "Oh, no! I don't need the money. I am saving that for my nephew Gottfried, whose father Fritz is now in Tomsk in Siberia. Gottfried will be a great mathematician someday, and he will certainly need the money." Emmy Noether had not changed.

After Noether had settled into life at Bryn Mawr, the question of what to do for her the next year came up. Funds had originally been arranged for one year only, and there was the continued problem of whether Bryn Mawr, a primarily undergraduate college for young women, was the best institution for her, since Noether clearly had no interest in teaching undergraduates. The Rockefeller Foundation required that such scholars move as quickly as possible into regular academic positions; something had to be done.

Arnold Dresden, the chair of the mathematics department at nearby Swarthmore College, solicited letters of support for Noether to be submitted to the Rockefeller Foundation. The responses were quick and positive.

Solomon Lefschetz at Princeton responded as follows:

> Professor Dresden has requested that I write to you regarding Professor Emmy Noether's place in the mathematical world. This will not take me very long; she is the holder of a front rank seat in every sense of the word. As the leader of the modern algebra school, she developed in recent Germany the only school worthy of note in the sense, not only of isolated work but of very distinguished group scientific work. In fact, it is no exaggeration

> to say that without exception all the better young German mathematicians are her pupils. Were it not for her race, her sex, and her liberal political opinions (they are mild), she would have held a first-rate professorship in Germany, and we would have no occasion to concern ourselves with her. She is the outstanding refugee German mathematician brought to these shores and if nothing is done for her it will be a true scandal.

To the same request, Norbert Wiener at MIT wrote:

> … Miss Noether is a great personality; the greatest woman mathematician who has ever lived; the greatest woman scientist of any sort now living, and a scholar at least on the plane of Madame Curie. Leaving all questions of sex aside, she is one of the ten or twelve leading mathematicians of the present generation in the entire world and has founded what is certain to be the most important close-knit group of mathematicians in Germany—the Modern School of Algebraists. Even after she was deprived of her position in Germany on account of her sex, race and liberal attitude, numbers of students (men as well as women) continued to meet at her rooms for mathematical instruction. Of all the cases of German refugees, whether in this country or elsewhere, that of Miss Noether should be without doubt the first to be considered.

Funds were found to provide two more years for Professor Emmy Noether at Bryn Mawr.

On April 7, 1935, Emmy Noether wrote a long letter to her friend and colleague Helmut Hasse discussing a variety of mathematical topics and joint publishing projects, and considering whether or not she would attempt to return to Germany again in the summer. At the very least, the letter continued, she would need to be present at the graduation ceremonies for Bryn Mawr at the end of June, decked out in full academic costume (academic cap and gown plus the hood signifying her doctorate). She was clearly looking forward to all the respect that her appearance as an academic marching in the somber ceremony would bring.

The next day, on April 8, 1935, Noether entered the hospital in Bryn Mawr, Pennsylvania, for surgery to remove an ovarian cyst reportedly the size of a cantaloupe. Two smaller uterine fibroid tumors were not removed because they were not causing her any discomfort and the physicians did not want the surgery to last any longer than necessary, although at Noether's request they did remove her appendix. Her letter to Hasse made no reference to her impending surgery—in fact, only a very few people knew anything at all about it. She apparently considered it little more than a minor inconvenience—she joked that it might give her a somewhat slimmer figure.

In the first four days after the surgery, she appeared to be making a fine recovery. Then, without warning, she fell into a coma and died. Although there is no formal report on the cause of death, one of her doctors noted that it could have been caused by a virulent infection that might have struck at the base of her brain (where he said the body's heat centers are located), since before she died her temperature reached an astonishing 109 degrees Fahrenheit.

Before the surgery, Noether had made a list of people to whom she would like to give a personal token if something unexpected should happen. Olga Taussky reported that she received a brooch that Olga had given Emmy and that Emmy had worn every day.

At the time of her death, Emmy Noether was 53 years old and at the height of her intellectual powers. She had not been a child prodigy; on the contrary, unlike most mathematicians, she did her first significant mathematics only after the age of 30. While many mathematicians have done great work at a very young age, others continue to do important work for many years, and a few (like Noether) start late and increase their productivity as they age. Nevertheless, her accomplishments are impressive. She is considered to be the greatest female mathematician the world has ever known and the mother of modern abstract algebra.

Tributes to the Mother of Modern Algebra

On April 26, 1935, Emmy Noether's colleague, friend, and fellow exile Hermann Weyl paid this tribute to her during a memorial service at Bryn Mawr, Pennsylvania:

> …. She was such a paragon of vitality, she stood on the earth so firm and healthy with a certain sturdy humor and courage for life, that nobody was prepared for this eventuality. She was at the summit of her mathematical creative power; her far-reaching imagination and her technical abilities, accumulated by continued experience, had come to a perfect balance; she had eagerly set to work on new problems. And now—suddenly—the end, her voice silenced, her work abruptly broken off.

As a mathematician, Weyl had tremendous respect for Emmy's completely original approach to mathematics:

> [Noether's] strength lay in her ability to operate abstractly with concepts. It was not necessary for her to allow herself to be led to new results on the leading strings of known concrete examples. This had the disadvantage, however, that she was sometimes but incompletely cognizant of the specific

> details of the more interesting applications of her general theories. She possessed a most vivid imagination, with the aid of which she could visualize remote connections; she constantly strove toward unification. In this she sought out the essentials in the known facts, brought them into order by means of appropriate general concepts, espied the vantage point from which the whole could best be surveyed, cleansed the object under consideration of superfluous dross, and thereby won through to so simple and distinct a form that the venture into new territory could be undertaken with the greatest prospect of success.

Weyl also felt great affection for Emmy as a person:

> It was only too easy for those who met her for the first time, or had no feeling for her creative power, to consider her [eccentric] … and to make fun at her expense. She was heavy of build and loud of voice, and it was often not easy for one to get the floor in competition with her. She preached mightily, and not as the scribes. She was a rough and simple soul, but her heart was in the right place. Her frankness was never offensive in the least degree. In everyday life she was most unassuming and utterly unselfish; she had a kind and friendly nature. Nevertheless, she enjoyed the recognition paid her; she could answer with a bashful smile like a young girl to whom one had whispered a compliment. No one could contend that the Graces had stood by her cradle; but if we in Göttingen often chaffingly referred to her as "*der Noether*" (with the masculine

> article), it was also done with a respectful recognition of her power as a creative thinker who seemed to have broken through the barrier of sex. She possessed a rare humor and a sense of sociability; a tea in her apartments could be a most pleasurable experience. But she was a one-sided being who was thrown out of balance by the overweight of her mathematical talent.

In summing up Emmy's life, Weyl wanted his audience to see the dynamic interplay of her awkward body with her brilliant mind in a world where major forces often worked against her:

> Indeed, two traits above all determined her nature: First, the native productive power of her mathematical genius. She was not clay, pressed by the artistic hands of God into a harmonious form, but rather a chunk of human primary rock into which he had blown his creative breath of life. Second, her heart knew no malice; she did not believe in evil—indeed, it never entered her mind that it could play a role among men. This was never more forcefully apparent to me than in the last stormy summer, that of 1933, which we spent together in Göttingen. The memory of her work in science and of her personality among her fellows will not soon pass away. She was a great mathematician, the greatest, I firmly believe, that her sex has ever produced, a great woman.

On September 5, 1935, Emmy's Russian colleague and friend P. S. Alexandroff gave a speech in Moscow with Fritz

Noether in attendance, honoring Fritz's sister. He concluded with these words:

> … a major woman! And this she was—her feminine psyche came through in the gentle and delicate lyricism that lay at the foundation of the wide-ranging but never superficial relationships connecting her with people, with her avocation, with the interests of all mankind. She loved people, science, life with all the warmth, all the joy, all the selflessness and all the tenderness of which a deeply feeling heart—a woman's heart—was capable.

In 1935, Emmy's former student and colleague Bartel Van der Waerden wrote a tribute to Emmy in *Mathematische Annalen*, the German publication for which she had edited and written extensively. Although it was risky, since she had been a Jew forced into exile from the Nazi regime, the *Annalen* boldly published it. Van der Waerden remembered her fondly as a teacher and mentor, beginning with the maxim that Emmy used as a guide to her mathematics: "All relations among numbers, functions, and operations become clear, generalizable, and truly fruitful only when they are separated from their particular objects and reduced to general concepts."

He continued,

> Her absolute, incomparable uniqueness cannot be explained by her outward appearance only, however characteristic this undoubtedly was. Her individuality is also by no means exclusively a consequence of the fact that she was an extremely talented mathematician, but lies in the whole structure of her creative personality, in the style of her

> thoughts, and the goal of her will.… Her thoughts were primarily mathematical thoughts, and the will [was] primarily intent on scientific recognition…. She could think only in concepts, not in formulas, and this is exactly where her strength lay. In this way she was forced by her own nature to discover those concepts that were suitable to serve as bases of mathematical theories…
>
> [Emmy Noether] had no didactical gifts, and the great pains she took to explain her remarks by quickly spoken interjections even before she had finished speaking were more likely to have the opposite effect. And still how exceptionally great was the impact of her talks, everything notwithstanding! The small, faithful audience, mostly consisting of a few advanced students and often just as many lecturers and foreign guests, had to exert themselves to the utmost to keep up. When that was done, however, one learned far more than from the most excellent lectures.

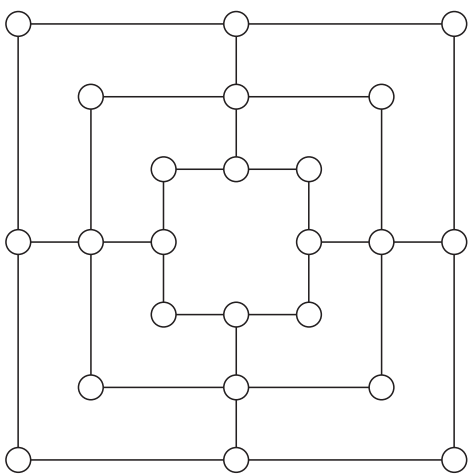
When Albert Einstein saw a very short notice (too short, he thought) of her death in *The New York Times*, he was outraged and wrote a letter to the editor on May 1, 1935:

> In the judgment of the most competent living mathematicians, *Fräulein* Noether was the most significant creative mathematical genius thus far produced since the higher education of women began. In the realm of algebra, in which the most gifted mathematicians have been busy for centuries, she discovered methods which have

> proved of enormous importance in the development of the present-day younger generation of mathematicians…
>
> Her unselfish, significant work over a period of many years was rewarded by the new rules of Germany with dismissal, which cost her the means of maintaining her simple life and the opportunity to carry on her mathematical studies. Farsighted friends of science in this country were fortunately able to make such arrangements at Bryn Mawr College and at Princeton that she found in America up to the day of her death not only colleagues who esteemed her friendship but grateful pupils whose enthusiasm made her last years the happiest and perhaps the most fruitful of her entire career.

In 1993, a planetoid that had been discovered in 1955 was named after Emmy Noether. The symbolism is apt. She continues to be a shining light.

Appendix
Nine Men's Morris



Nine Men's Morris is a game played throughout Europe in public parks, cafés, homes, and neighborhoods.

Two players each have nine markers—they may be two different colored bingo chips, two different kinds of beans, or pebbles and sticks—which are placed one at a time on the circles at the intersections of lines. In the game, each player tries to form mills with her own markers. A mill is formed by three of the same markers placed on the circles, making a horizontal or vertical straight line. If two mills intersect, they may

both use the same marker at the point where they intersect. A mill consists of only one kind or color of marker.

Round one: The players take turns, each placing one marker on any open circle on the grid. Each player tries to make mills and to interfere with the opponent's mills. Any marker that is part of a complete mill is safe—the opponent may not remove it. Whenever a player makes a mill, he removes one of his opponent's markers, but it must be a marker that is not part of a mill. Each piece that is removed is out of play for the rest of the game. This round ends when all the pieces have been placed on the grid.

Round two: Players alternate, with each player moving one of her own markers one step in any one turn in an attempt to form a mill and/or to stop the opponent from forming mills. A player may move a piece only through open circles, along the lines, and she must move one piece one space on each turn. Sometimes a player can set it up so that she has two parallel mills formed, enabling her to move one center piece back and forth between the two mills. Each time she makes a new mill, she may remove one of the opponent's pieces that is not part of one of his mills. She is allowed to reform the same mill repeatedly, taking one of her opponent's pieces each time.

The end game: There are two ways a player can win. One way is when his opponent has only two pieces left on the board, meaning that the opponent can no longer make a mill. The other way to win is if the opponent cannot move any pieces because they are all blocked.

Glossary

Abitur. The examination required for a high school diploma in Germany.

Algorithm. A method for solving a problem using a specific set of steps.

Axiom. A statement that is assumed without proof.

Axiomatic. Based on axioms; an approach to a (mathematical) theory based on a set of axioms and rules of deduction.

Bratwurst (plural Bratwürste). A grilled sausage.

Cardinality. The "number of members" of a set.

Commutative. The property of addition or multiplication stating that changing the order of quantities does not change the total (ex. ab = ba or a+b = b+a).

Deutsche Mathematiker-Vereinigung (DMV). The German Mathematical Association.

Diophantine equation. An equation in which only integer solutions are allowed.

Doktormutter. Female mentor or doctoral thesis advisor (literally: doctor mother).

Doktorvater. Male mentor or doctoral thesis advisor (literally: doctor father).

Euclidean Geometry. A geometry based on the work of Euclid of Alexandria (300 BC) consisting of definitions, axioms, and theorems. In Euclidean geometry, given a straight line and a point not on the line, there exists one and only one straight line which passes through that point and never intersects the first line.

Extraordinarius. Assistant professor (literally: extraordinary professor) who is allowed to mentor PhD students.

Field. A set (such as the set of rational numbers) in which certain operations such as addition, subtraction, multiplication, and division are defined and obey the familiar set rules known from the rational numbers.

Frau. Mrs.

Fräulein. Miss.

Fundamental theorem of algebra. A statement that a polynomial equation in one unknown has as many roots as its degree (the largest exponent in the variable).

Gänseliesel. The goose-girl fountain in front of the city hall in Göttingen.

Grundschule. Elementary school.

Gymnasium. College-preparatory high school.

Habilitation. The process by which a German scholar acquires the right to teach at a university, through the production of a large scholarly work and its approval by his university.

Herr. Mr.

Höhere Töchterschule. A privileged girls' school (advanced school for the daughters of the town's elite families).

Ideal. A subset of a ring that is closed under addition and multiplication, meaning that operations such as adding and/or multiplying within the set always result in a total that is also a member of the set.

Identity function. A calculation that results in no change, like adding zero or multiplying by one.

Invariant. A property of an expression that does not change when the expression is transformed (for example in a circle, no matter how big or small it is, the ratio of its circumference to its diameter is invariant—it is always π).

Inverse function. A calculation that reverses another calculation. In multiplication, multiplying by the reciprocal is the inverse function.

Matrix (plural Matrices). Sets of quantities arranged in rows and columns allowing a convenient process to perform operations with these quantities.

Non-commutative. The property of an operation in which changing the order gives a different result. Subtraction and division are non-commutative since $8 - 3 \neq 3 - 8$ and $8 \div 3 \neq 3 \div 8$.

Non-Euclidean Geometry. An axiomatic system of geometry that is Non-Euclidean.

Ordinarius. A full professor in a German university (literally ordinary professor). An *Ordinarius* has higher status than an *Extraordinarius*.

Privatdozent. A lecturer at a university who is not allowed to serve as a mentor to PhD students.

Property. A characteristic of an object or set. The commutative property of addition states that the sum remains the same when the order of the terms is changed.

Real Gymnasium. A high school that includes modern languages, mathematics and science, but no instruction in Greek.

Reichsgesetze. Imperial law.

Reichsmark. The name for German money (1871–1923 and again 1925–1945).

Reichstag. German parliament (1871–1945).

Rentenmark. Temporary German money (1923–1925), introduced to stem hyperinflation.

Ring. A set with two commutative and associative binary operations (like addition and multiplication), identity operations that leave the quantity unchanged (like adding zero or multiplying by one), and an inverse operation that returns a quantity to the identity (like adding the opposite).

Stadtarachiv. City archive.

Tante. Aunt.

Topology. A branch of mathematics that studies invariants of shapes and space during continuous transformations such as stretching and bending without breaking or punching holes (sometimes called rubber sheet geometry, referring to the result when you stretch or twist a flexible surface).

Index